

Middle School Mathematics Task Force Subcommittee Report

Name of Subcommittee: Numbers and Operations/Rational Number and Proportional Reasoning
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Current Course Overview:

Numbers and Operations Course

The course is designed to develop a comprehensive understanding of our number system and how its structure is related to computation and of the properties that form the foundation for algebra. Special attention is also given to children's thinking, how they learn the fundamentals of number systems, their problem solving strategies, and how they construct their understanding of various number systems and arithmetic.

Rational Number and Proportional Reasoning Course

This course is designed to engage participants in constructing relational understanding in the theoretical development of mathematics and students' learning of mathematics within the content strands of rational numbers and proportional reasoning. There is also a focus on the development of pedagogical content knowledge of rational numbers and proportional reasoning appropriate for K-8 and on the assessment of K-8 students' mathematical conceptions and disposition towards mathematics (i.e., knowledge, beliefs, attitudes, and emotions) through interviews.

Current Goals of the Course:

Numbers and Operations Course

1. Exploration and a thorough development of number concepts and skills associated with
 - the number system and place value;
 - the meaning and interpretation of the arithmetic operations of addition, subtraction, multiplication, and division;
 - ratios, fractions, and decimals.
2. Study of how children learn arithmetic and the development of their understanding, including especially the use of the *Developing Mathematical Ideas* materials on number and operations.
3. The use of concrete activities to develop number and arithmetic concepts.
4. Development of the mathematician's patterns of behavior and habits of mind.

Rational Number and Proportional Reasoning Course

1. The participant will deepen his or her own understanding of rational numbers and proportional reasoning.
2. The participant will develop his or her professional competence, confidence and enthusiasm for teaching and learning mathematics.
3. The participant will deepen her or his understanding of children's mathematical learning of rational numbers in grades K-8, in the spirit of the NCTM reform documents.

4. The participant will develop teaching strategies for planning and teaching mathematics; that is, knowing how to select appropriate mathematical tasks, asking productive questions, giving useful explanations, and evaluating student work.
5. The participant will learn how to select and use manipulatives and pictorial representations to connect the concrete phase of mathematics learning to the abstract, symbolic phase of learning.
6. The student will become knowledgeable about the recommendations of various members of the mathematics education community for content and curricular knowledge of school mathematics.
7. The student will take responsibility for her or his own professional growth and contribute to the field of mathematics education.

Rationale for adaptations to accommodate the needs of Middle School Specialists:

A deep understanding of numbers requires teachers to extend their knowledge of number theory through rational numbers in order that they have multiple strategies for examining a problem scenario. Middle School Specialists will be working in schools where many teachers are possibly weak mathematically and pedagogically and where many students are ill prepared to learn the mathematics in the curriculum. Therefore, these specialists need a deep understanding of fractions, decimals, percents, proportions, rates and ratios, the derivation of measurement formulas and the content of Algebra 1. They also need to know how to assist teachers in developing approaches for teaching reasoning and proof to their students. Specific examples of the knowledge needed at the middle school level follows. Teachers should be able to explain, discuss and illustrate:

- Fraction concepts and skills including:
 - Work with mental math with fractions
 - Fraction benchmarks
 - Division with zero
 - Division with fractions
 - Fractions looking towards algebra
- Fraction instructions should look towards algebra and have a focus on generalizations and discussions that lead, for example, from $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ to an understanding of $\frac{1}{x} + \frac{1}{y} = \frac{x + y}{xy}$.
- The importance of the use of the real number line
- Long division with decimals
- Generalizations with rational numbers such as:
 - How to explain $\frac{1}{2} + \frac{1}{3}$ and then to generalize the concept to $\frac{1}{x} + \frac{1}{y}$
- The reasoning behind solving:
 - $\frac{3}{4} - \frac{1}{5}$
- The algebraic reasoning integrated throughout the math.
 - What are the experiences in arithmetic that lead to algebra?
 - Use of stories, narratives, number lines, and multiple representations

- Focus on generalizations
- Move towards using whole and natural numbers to build rational numbers (density property)
- Look at prime numbers and then decomposing them
- Stress the importance of proportions with fractions and negative numbers
- Move from rational numbers course to algebra should include operations with real number such as $1/\pi$

Recommended Course Resources to support Middle School Specialists:

Connected Mathematics, Glenda Lappan, 2006

Fundamental Constructs in Mathematics Education, Edited by John Mason and Sue Johnston-Wilder

Improving Instruction in Algebra, Margaret Smith, 2005

Improving Instruction In Rational Numbers and Proportionality, Margaret Smith, 2005

Teaching Fractions And Ratios For Understanding, Susan Lamon, 2005

Recommended Course Adaptations:

The current Numbers and Operation course develops a comprehensive understanding of our number system and how its structure is related to computation, and of the properties that form the foundation for algebra at the K-5 level. The Rational Numbers and Proportional Reasoning course engages participants in a theoretical development of mathematics and students' learning of mathematics within the content strands of rational numbers and proportional reasoning. The recommendation of this committee is that more is needed at the middle school level to address a deeper understanding of fractions and the connections to algebra with an emphasis on generalizing. The course needs to guide Math Specialist in knowing how to promote an understanding of fractions, decimals, percents, and proportions by having students generalize from their conceptions of problem situations that they experience as meaningful.

Summary Comments:

Since the mid 1960s, school divisions have struggled to create a developmentally appropriate learning environment that meets the needs of middle school students. In addition, the Carnegie Council for Adolescent Development (1989) reported that middle schools need to focus "on the characteristics and needs of young adolescents" and to create "a community of adults and young people embedded in networks of support and responsibility that enhance the commitment of students to learning". The overriding question is how to meet the needs of middle school students and how to support teachers

in presenting an engaging and coherent mathematics curriculum. In the context of Numbers and Operations/Rational Number and Proportional Reasoning courses, we have the added task of presenting a middle school curriculum to students who often are not prepared with an understanding of fractions and decimals. More needs to be done to fully prepare the Math Specialist at the middle school level to meet these challenges.