K-5 MATHEMATICS SPECIALISTS' TEACHING AND LEARNING ABOUT FRACTIONS

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Abstract

This paper describes the fraction-based mathematical activities of two teachers who are part of a Mathematics Specialist preparation program. Their work with fractions is traced from two perspectives: 1) their interactions with students as they struggle with fraction concepts; and, 2) their personal journeys to develop deeper understandings of fractions as participants in the *Rational Numbers* course that is part of their degree program. Through their stories, we gain a better understanding of the complex nature of their work with students and how their participation in the Mathematics Specialist program helps support their work in the school buildings.

Introduction

Our first cohort of graduate students has recently completed a master's degree program slated for Mathematics Specialists. Upon completing this degree program, these students are also eligible for a state licensure Mathematics Specialist endorsement. This endorsement is part of an effort to place one Mathematics Specialist in Virginia's K-8 schools for every 1,000 students—an initiative recommended by the State Board of Education. This initiative is not yet a funded recommendation. This move toward a K-8 Mathematics Specialist program is long awaited and is the result of over two decades of statewide efforts spearheaded by the Virginia Mathematics and Science Coalition (VMSC), a collaborative venture among district, university, and K-16 education stakeholders.

What is the Mathematics Specialists' role in the elementary school building? The list of responsibilities is long and appears to be growing as we consider recent proposals by mathematicians, mathematics educators, and organizations like the Virginia Mathematics and Science Coalition [1-4]. Reys and Fennell, for instance, describe the Mathematics Specialists' role using two models: lead-teacher model or the specialist-teaching-assignment model [3]. When the Mathematics Specialist serves in a teacher leader role, he or she is "released from classroom instruction to assume mentoring and leadership responsibilities at the building or district level" [3, 5]. One might expect a Mathematics Specialist to plan, co-teach, make observations, model lessons, and so on [3]. By way of contrast, Mathematics Specialists that serve in the specialist-teaching-assignment role assume, for instance, the primary responsibility

for teaching mathematics at a particular grade level [3]. A fifth grade Mathematics Specialist might teach math to all of the fifth graders, as well as provide professional development for the vertical math team (i.e., third, fourth, and fifth grade teachers of mathematics).   Reys and Fennell suggest that in the latter case, the classroom teacher develops a more narrow set of competencies and responsibilities [3].

The Virginia Mathematics and Science Coalition (VSMC), too, offers their own vision of the Mathematics Specialist's role:

> [K-8] Mathematics Specialists are teacher leaders with strong preparation and background in mathematics content, instructional strategies, and school leadership. Based in elementary and middle schools, Mathematics Specialists are former classroom teachers who are responsible for supporting the professional growth of their colleagues and promoting enhanced mathematics instruction and student learning throughout their schools. They are responsible for strengthening classroom teachers' understanding of mathematics content, and helping teachers develop more effective mathematics teaching practices that allow all students to reach high standards, as well as sharing research addressing how students learn mathematics [6].

As the VSMC suggests, the Mathematics Specialist assumes responsibility for promoting and supporting professional growth for their colleagues that lead to supporting or enhancing student learning.

The characteristics outlined in these definitions of a Mathematics Specialist have informed our work with teachers. The program has as its goal to support the transition of Mathematics Specialists into roles that parallel the description offered by VSMC. In addition to following Reys and Fennell, we hope that, ideally, graduates from the program would acquire positions that fit with the lead-teacher model [3].

For the last few years, we have made a concerted effort to understand the Mathematics Specialists' roles in different school settings as they become or continue to serve as Mathematics Specialists. As part of this process, we have followed six of the twenty-six participants in the first cohort in this degree program. To document their activities, we videotaped all of the class meetings for three of the five mathematics courses and two of the three education leadership

courses that are part of the graduate degree program. In addition, we made visits to their school buildings each of the three years that they participated in the program. During our school-site visits, we also conducted audio taped interviews to address aspects of their daily work. By collecting these different types of information, we have attempted to understand how their participation in this graduate program has supported, in part, their work with teachers and their students. This article is our first attempt to develop a report that coordinates their experiences in the degree program with their work in the school buildings.

Ms. Smith and Ms. Sneider

To better understand how their experiences in the degree program might support the participants' daily work in schools, we use examples taken from both sets of data: their school-based activities and their participation in course activities. In our discussion, we use examples taken from our school-site visits at two of the participants' school buildings to illustrate how they use mathematics in their daily work. We then highlight an example from one of their class discussions in the course entitled, *Rational Numbers and Proportional Reasoning*, one of the mathematics courses in their degree program. Here, we tell the story of two of our recent graduates, "Ms. Smith" and "Ms. Sneider." Ms. Sneider's responsibilities are similar to those described by the leader-teacher model. She serves as a Mathematics Specialist in her school building. By way of contrast, Ms. Smith's responsibilities align more with the specialist-teaching-assignment model—she is a regular classroom teacher. As we tell parts of their stories, we attempt to understand what their roles might entail and how their roles are supported through their participation in the Mathematics Specialist program.

In both of our school-based examples, Ms. Smith and Ms. Sneider worked with similar concepts related to students' beginning understanding of fractions. Ms. Smith's example is taken from an introductory fraction lesson that she co-taught with another teacher while Ms. Sneider's example is taken from a lesson that she taught to a small group of fifth graders. We first provide examples of their daily work and then we make connections between Ms. Smith's and Ms. Sneider's graduate course experiences with fractions and their leadership roles in their respective school buildings. We begin our discussion by telling part of Ms. Smith's story.

Background—"What Fractional Part of the Two Pizzas is Left?"

Ms. Smith currently teaches fourth grade and is responsible for all instruction in all subject areas. Prior to the 2006-07 school year, Ms. Smith taught at a school where she had been

a primary grade teacher for six years. Ms. Smith was one of the Lead Teachers for mathematics and science instruction in her building. She also worked closely with the building math coach (i.e., Mathematics Specialist). She, in fact, hoped to serve in a similar role once she completed the Mathematics Specialist program. After completing her first year in the program, Ms. Smith was reassigned to a different school building for the 2006-07 school year. In addition to teaching in a different school building, she was assigned to a new grade level—fourth grade. Ms. Smith had never taught fourth grade before.

One of the ways that Ms. Smith capitalized on leadership opportunities as a fourth grade teacher was through co-teaching mathematics with "Ms. Applebee," a special education teacher. To our surprise, these two teachers did not know each other before they began working together. As Ms. Smith commented once during an interview, "We did not know each other from a hill of beans." One would not have suspected that they had never worked together. During our first visit to their classroom, we realized they had developed a rich, collaborative, professional relationship.

Ms. Smith and Ms. Applebee often met before or after school to plan mathematics lessons. They frequently exchanged ideas about how they would introduce the lesson, which students might need additional support, what activities they would use, etc. Both teachers stood in the front of the room during whole class discussions, and moved from group to group during independent or small group work. Usually, Ms. Smith introduced lessons and orchestrated whole class discussions although Ms. Applebee, too, helped lead discussions.

The Lesson—"What Fractional Part of the Two Pizzas Is Left?"

Our example is taken from an introductory lesson we observed about adding fractions. For this lesson, students solved the following problem independently: "Patrick ate 1/8 of a pepperoni pizza and 3/8 of a cheese pizza. How much pizza did he eat?"

After the students solved this and several other problems, Ms. Smith led a whole class discussion about the above problem. She began the discussion by asking the students what equation they had written to represent this problem. She then asked the students why they decided to combine the two fractional parts to determine what Patrick had eaten.

After students agreed that Patrick had eaten 4/8 of a pizza, Ms. Applebee asked the students why the answer was not 4/16 instead of 4/8 of a pizza. When Ms. Applebee asked this

question, the students became very quiet. Previously, the students had engaged in a lively discussion about why the answer was 4/8 (see Figure 1).
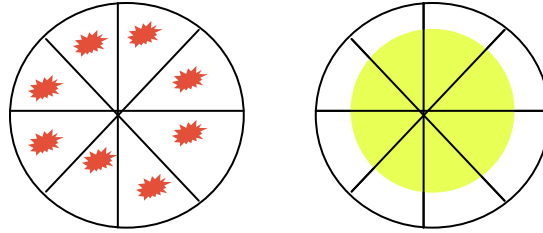

Figure 1. Ms. Smith draws one pepperoni and one cheese pizza.

When Ms. Applebee asked why the answer was not 4/16, the students seemed puzzled. When none of the students attempted to answer Ms. Applebee's question, Ms. Smith referred to the pictures of pizzas on the board and asked a different question. She asked the students if they could make one whole pizza with the remaining pieces of pepperoni and cheese slices (see Figure 2).
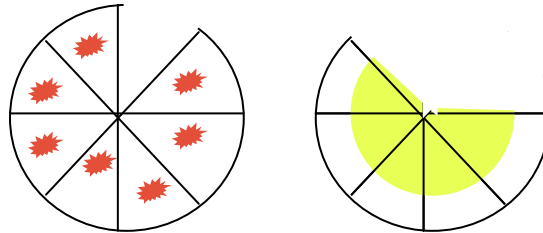

Figure 2. One slice of pepperoni and three slices of cheese pizza are missing.

In response to Ms. Smith's question, students explained how they would move three of the leftover pepperoni slices to the cheese pizza to make a whole pizza. Using both pepperoni and cheese slices, they would then have one whole pizza and one-half of a second pizza remaining. Ms. Smith recorded the students' ideas using arrows and drawing three slices to fill up the cheese pizza (see Figure 3). She also wrote the fractional amounts under each pizza (see Figure 4).
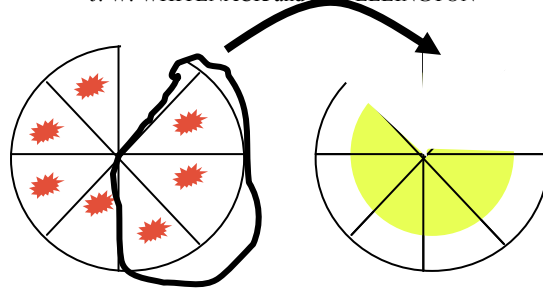
Figure 3. Ms. Smith represents moving three pepperoni slices to make one whole pizza.
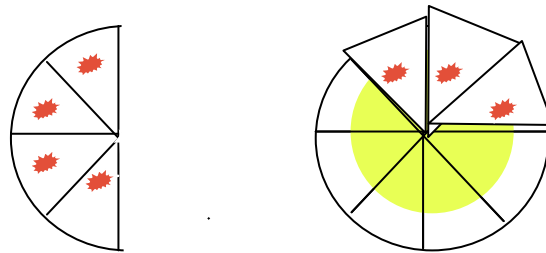
Figure 4. Ms. Smith represents one whole pizza and one-half of a pizza to illustrate the number of slices that remained.

Discussion—"What Fractional Part of the Two Pizzas Is Left?"

As the lesson unfolded, we wondered why Ms. Applebee asked this question at this juncture. Had she spoken with students who had derived this answer of 4/16 instead of 4/8 for the answer? Or did she hope to engage the students in a discussion about a common error that she has seen other students make when they combine fractions? We also wondered how Ms. Smith might orchestrate the discussion following Ms. Applebee's question. From above, we know that Ms. Smith chose not to address Ms. Applebee's question during this lesson. Instead, she asked the students a different question that refocused the discussion around combining fractions with like denominators. Her question proved to be an important one. By asking this question, students had an opportunity to explore ideas related to making whole pizzas (units) with the remaining slices (eighths).

As she initiated this teacher move, she also indirectly supported Ms. Applebee's teacher moves during this part of the lesson. Although Ms. Applebee's question is an important one for the students to consider (at some point during this fractions unit), Ms. Smith's decision to redirect the discussion was an important teacher and coaching move. As Ms. Smith asked this question,

she was also in the position to support Ms. Applebee as she made contributions during the lesson. When Ms. Applebee asked a question that did not appear to move the students' thinking forward, Ms. Smith could offer a different question so that students could consider some related, important ideas about combining fractions. As such, this situation was a possible learning opportunity for the students, as well as for Ms. Smith and Ms. Applebee. By redirecting the question, students had the opportunity to use ideas to explore another problem involving addition with fractions. Ms. Applebee had the opportunity to "see" a possible teaching move that might be more appropriate at this point in the unit about fractions. In order to facilitate this shift in the discussion, Ms. Smith drew on those mathematical ideas that she understood about fractions to address a situation that she had not anticipated prior to this lesson.

During our taped debriefing session following the lesson, we asked Ms. Smith why she decided to ask the question about combining the leftover pieces of pizza. Ms. Smith explained:

> And so I think that is where I was trying to bring them back to. "So if you have pepperoni pizza…Can you re-form that whole? Does it change how many pieces that whole is cut into?"

Ms. Smith chose to move the discussion forward by relating the problem to ideas that the students had previously explored. Two ideas that she hoped to address were reforming the whole and conserving the whole or what she referred to as "chang[ing] how many pieces."

Without prompting, she then related her students' thinking to ideas that she had encountered in the *Rational Numbers* course that she had successfully completed the previous summer:

> The students' thinking is amazing to me. It is amazing to me—the idea of the parts and what makes up the whole… Some of the same things we were dealing with this past summer in our own [*Rational Numbers*] class.

As we pursued the influence of the course on her teaching, she offered additional insight into how her instructional approach had changed:

> Oh, yeah [laughing]. I would be there right with them. "Okay, let's multiply by two and get a common denominator..." I wouldn't have had a clue as to how to teach this math topic. I would have had the textbook out, and I would have used a little bit of *Innovative Mathematics* and I would have said, "I don't know how I am going to

get from here to here." …And I see a little bit as to how we will get to those places…

She viewed her experiences in the course as important because she could "see a little bit as to how we will get to those places"—places they needed to reach as she supported her students' understanding of fractions. Rather than simply following the curriculum as presented in her teacher's guide, she could initiate discussions around some of the important ideas about fractions. So, Ms. Smith's work in the course contributed, in part, to how she could better teach ideas around fractions. We also suspect that her experiences in the course made it possible for her to offer potential situations for coaching Ms. Applebee about teaching their ideas more effectively.

We now turn our attention to Ms. Sneider's work as a Mathematics Specialist.

**Background—"What Fractional Part Is the Yellow Pattern Block?"**
Ms. Sneider is a full-time Mathematics Specialist in her school building. She, too, had successfully completed *Rational Numbers* during the previous summer. As a Mathematics Specialist, one of the challenges she faced was scheduling time to visit with teachers at each grade level throughout the school year. As part of her plan, she worked with teachers in a particular grade level for several weeks, and then moved to another grade level to work with a different group of teachers. As she worked with teachers, she sometimes co-taught lessons or made drop-in visits to classrooms while teachers were teaching mathematics lessons. When she made drop-in visits, it was not uncommon for her to interject comments during the lesson. When students completed assigned problems as they worked independently or in small groups, she typically walked around the room, stopping at an individual student's desk to ask clarifying questions, listening to the student's explanation or, in some cases, providing additional instruction.

During her second year as a Mathematics Specialist, she also worked with small groups of students who were pulled out of their classrooms to receive additional support. Our example is taken from one of these pullout sessions. In this particular pullout session, Ms. Sneider worked with a small group of fifth graders who continued to struggle with understanding fractions. The fifth grade teachers asked her to work with these students to prepare them for the upcoming school building quarterly assessment—a benchmark assessment in preparation for the statewide mathematics test.

The Lesson—"What Fractional Part Is the Yellow Pattern Block?"

Ms. Sneider began this session by asking students to make a yellow hexagon shape (the unit) using other pattern blocks. Pattern blocks are six geometric shapes: green equilateral triangles, blue rhombuses, tan rhombuses, orange squares, red trapezoids, and yellow hexagons (see Figure 5). Red, green, and blue blocks can be used to make yellow blocks. The green blocks can be used to make blue blocks or the red blocks, etc. As each of the students explained their pattern block configurations, they seemed confused about what fractional part each of the six green triangles represented. Although some of the students stated correctly that one green triangle represented 1/6 (e.g., because six green triangles made one hexagon), it was not clear if students understood that these six triangle pieces needed to be the same size. To address this misconception, Ms. Sneider made a different shape using all 6 shapes (see Figure 5). She referred to this configuration as a "funky cookie."
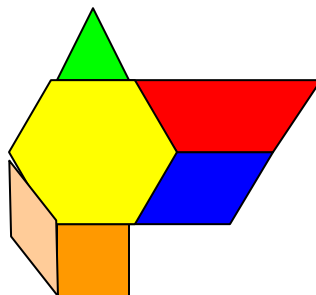
Figure 5. Ms. Sneider makes a "funky cookie" using all six pattern blocks.

After Ms. Sneider made this funky cookie, she asked the students what fraction the yellow hexagon block represented. Not surprisingly, students were not sure what this fractional part was. She then asked if she could share the cookie fairly by giving each student one of these six pieces. Following her questions, the students stated that if she shared her funky cookie, she would not share her cookie fairly. After some discussion, several students made different shapes using the blue and green blocks and correctly explained how they could share their pieces fairly by divvying out blocks so that each person could receive the same amount.

Discussion—"What Fractional Part Is the Yellow Pattern Block?"

As we observed this session, we were not aware that Ms. Sneider had decided to change her lesson plan. As she explained later during our debriefing session, she realized that the students did not necessarily understand that each of the 1/6 needed to be the same size. The students understood that they needed six pieces to make the whole, but that they did not

understand that those pieces needed to be the same size. Once she realized that they did not have a solid understanding of what constitutes fractional parts, she decided to scrap her original lesson plan—helping students change improper fractions to mixed fractions (e.g., $5/3 = 1\frac{2}{3}$) using pattern blocks. Instead of introducing a new activity, she posed several tasks in which students used pattern blocks to make the whole.

Her decision to pose the "funky cookie problem" was a critical point in her revised lesson. Her decision to make a pattern block configuration that involved *unequal* pieces was a particularly important one because it explicitly highlighted the misconception that the students had about fractional parts.

Observations

Both of our examples illustrate how Ms. Smith and Ms. Sneider used their understanding of key mathematical ideas to support their students' reasoning about fractional parts. Interestingly, although they had not planned to pose these particular problems, they both made important, on-the-fly decisions that advanced their instructional goals. They used their understanding of the mathematical ideas related to fractions in unique ways as they worked with their students.

One of the reasons that they were able to do so was because of their experiences in *Rational Numbers*, a course that they had completed during the previous summer. Recall that Ms. Smith actually referred to the importance of *Rational Numbers* in the debriefing session. Ms. Sneider, too, mentioned during debriefing sessions that her experiences in *Rational Numbers* were part of the reason she could pose these types of tasks, tasks that challenged students to think about important ideas about fractions. So what opportunities did participants have to explore and build new ideas about fractions? To answer this question, we turn to our example from the course.

**Exploring Rational Numbers—"Can You Find a Fraction between 1/11 and 1/10?"**

To illustrate the types of experiences that they had during *Rational Numbers*, we highlight part of one of the lessons that occurred during the second week of the course. For this lesson, participants explored an activity from "Bits and Pieces: Part I," one of the fraction modules from the *Connected Mathematics* curricular series [7]. To begin this lesson, the course instructor asked participants, in small groups, to find a fraction between 1/11 and 1/10.

Participants had solved a similar problem for homework (i.e., "Can you find a fraction between 1/10 and 1/9?").

Exploring Rational Numbers—The Lesson

To introduce this problem, the course instructor drew a fraction strip and as the discussion ensued, he explained how he could use the fraction strip to represent these different fractions (see Figure 6):

> And remember that we were working with these strips—fraction strips. We were looking at those fraction strips [draws a picture of a unmarked fraction strip on the white board] and marking them so that by folding first here, we have a ½ [makes a mark and writes ½, and divides it into fourths]. And this of course would be 2/4 [writes these numbers on the fraction strip]… The rational numbers there are representing distances from 0. So that's one way—a very, very natural way that rational numbers appear as distances. Remember that we extend them so that it went beyond 1 [extends the fraction strip and writes 1 at the hash mark that represents 4/4].
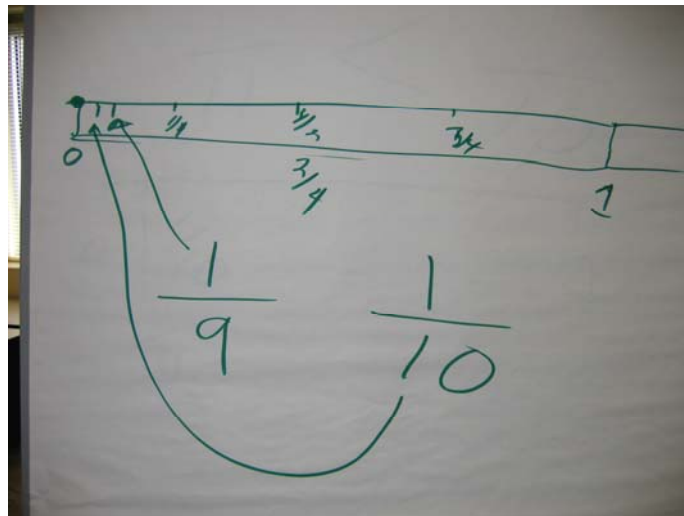
Figure 6. The instructor used the fraction strip to represent ¼, ½, ¾, and 1.

As the discussion continued, the course instructor marked approximately where 1/10 and 1/9 were located on this strip (see Figure 7). After marking these numbers on the number strip,

he asked the participants if they could think of a fraction that was smaller than 1/10. Several participants, in unison, said that 1/11 was a fraction that was smaller than 1/10.
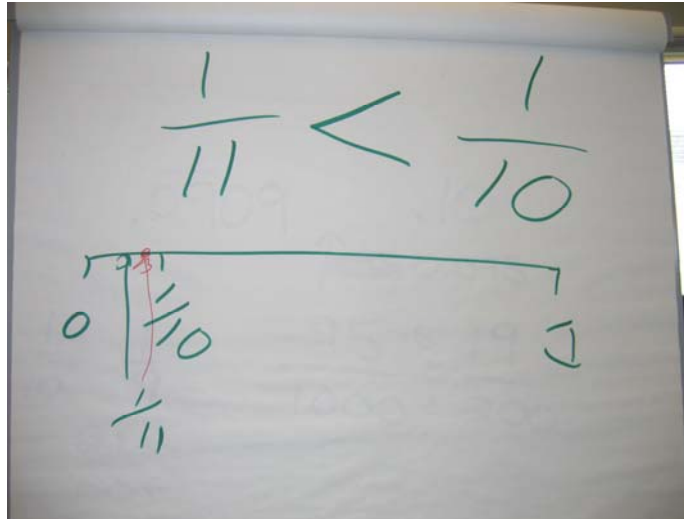
Figure 7. The instructor used the fraction strip to represent 1/11 and 1/10.

As the discussion continued, he posed the problem that they would explore in their small groups:

There are lots of numbers that are less than 1/10, but one that is nice, that is less than 1/10 is 1/11. Just to get ourselves going again, at each of the tables, figure out a way to find a rational number between 1/10 and 1/11...Then I'll ask you to come up and share with us.

Participants began to work with others sitting at their tables to devise or refine their methods for finding fractions between 1/11 and 1/10.

Ms. Smith and Ms. Sneider worked with two other participants at their table. Ms. Sneider talked at some length about one participant's method. Ms. Smith used Ms. Sneider's approach to find other fractions. As we asked questions about their solution methods, Ms. Sneider explained her ideas about finding a fraction between 1/10 and 1/9, the homework problem:

[T]he other night when I figured out this problem. I thought, oh, I finally found a fraction between these two [fractions]. And then I let it rest. And then we come here; we talked about it and everything. Well, I couldn't get that problem off my

mind, so I was thinking about it more over the weekend, and I finally thought to myself, "What if I didn't [multiply by] 2, what if I multiplied by 3?" Then I'd have 3/30, and 3/33. And there'd be two fractions… 31sts and 32nds that could go. Then I thought, "What if I multiplied it by 4?" And so you can multiply it by anything. So it gets you close to—if you kept on going—that there are an infinite number [of fractions]. But that was an "aha" moment when I realized that you can do it with more than just [multiplying by] 2!

As her comment suggests, Ms. Sneider figured out that she could generate equivalent fractions by multiplying the numerator and the denominator by the same number. In fact, she claimed that she could find an infinite number of these fractions between 1/10 and 1/11. When Ms. Sneider made this comment, Ms. Smith nodded her head in agreement.

We also talked with Ms. Smith about her method for finding fractions. Ms. Smith explained that she multiplied both 1/10 and 1/11 by 4/4 to rename them as 4/40 and 4/44. As she explained her answer, she pointed to Ms. Sneider as if to indicate that she had decided to use Ms. Sneider's method to find this fraction:

> I just wanted to see if I could do this a different way [points to Ms. Sneider]. So I tried 4 over 42; that is what I did… So I just split 4 and 42 and it still reduced down to 2/21.

So Ms. Smith used a method similar to the one that Ms. Sneider had used to find fractions between 1/10 and 1/9. The first part of her comment, "I just wanted to see if I could do this a different way" is curious. Had she initially solved the problem differently? As it turns out, she had. For her first attempt at this problem, she had used a calculator to rename each fraction as its decimal equivalent, and then had found a decimal that was larger than $.09\overline{09}$ and smaller than $.1000$. She used Ms. Sneider's method to find the result after she had used the decimal method. So she used Ms. Sneider's method to experiment with a different method.

To begin the whole class discussion, the course instructor asked one of the participants to share her method with the class. Like Ms. Smith, this participant shared that her group converted 1/10 and 1/11 to their decimal equivalents. She explained that 1/10 was equivalent to 0.1000 and

1/11 was equivalent to the repeating decimal .09090… So, .095 (or 95/1000) was one of the fractions between 1/11 and 1/10. After this participant shared this idea, Ms. Sneider suggested, without prompting, that she could have also chosen .091, .092, .093,…or .099. She then argued that to find a decimal (and its fraction equivalent), one merely needed to increment the digits, in this case, in the thousandths place. She then related this strategy to how one incremented the digits to manipulate whole numbers—92 is one more than 91, 93 is one more than 92, etc.

As the discussion continued, another participant shared her group's method for finding other fractions. She explained that she first converted 1/11 to 10/110 and 1/10 to 11/110. Then, she stated that 10½/110 was halfway between 10/110 and 11/110. She demonstrated this fact by drawing an open number line and marking 1/11 and 1/10 on this number line. She then drew a line halfway between these two fractions and indicated that this mark on the number line was the position of the fraction that they had found. At this point in the discussion, the course instructor turned to the whole class and asked a question about this group's method. As he did so, he again referred to the fraction strips:

Instructor: Before you go any further there, if you have one of these fraction strips, how many pieces would it fold up into now?

Participants: [In unison] 110.

Instructor: 110 pieces. Can you go from actually folding 8 or folding 12, to actually thinking in your mind 110 folds? I couldn't do 110 folds; I'm not that good. But I kind of think it's as if I had folded 12 times. It's the same idea. So it's folded into 110 little pieces.

As the discussion continued, the participant explained that her group struggled with how to represent 10½/110. Because they did not like how their new fraction was written (i.e., it was an improper fraction), they split each 1/110 and created smaller pieces that were one-half of 1/110, 1/220.

Again, the instructor asked clarifying questions about how this group generated these smaller pieces. He first asked if her group had folded (or imagined folding) each piece in half. After responding again that they would have 220 pieces, the participant then explained that after splitting each piece in half, they could rename 10/110 as 20/220 and 11/110 as 22/220. By renaming 10½/110 as 21/220, they took care of their "problem" of working with improper fractions. So 21/220 was one proper fraction that they found that was between 1/11 and 1/10.

As the whole class discussion continued, several other participants explained how they used different methods to find fractions between 1/11 and 1/10. Another group, for instance, renamed 1/11 and 1/10 as 3/33 and 3/30. They then explained that they could find two fractions between these two fractions, 3/32 and 3/31. To justify their answer, they explained that their strategy was similar to when one orders the unit fractions, ½, 1/3, ¼, 1/5… To find a small fraction, they simply needed to increment the denominator as long as each of these fractions had the same numerator.

As the discussion ensued, the course instructor clarified participants' explanations and asked questions to check for the participants' understandings. Throughout the lesson, participants had opportunities to understand others' methods for finding fractions between two given fractions. As they did so, they began to explore the density property, one of the important properties that is unique to the set of Rational Numbers (and Real Numbers).

Exploring Rational Numbers—Discussion

At the outset of this lesson, we see that the course instructor used a different approach to introduce ideas—an approach that seems quite different from a more traditional lesson about ordering fractions. The course instructor, for instance, referred to different fractions as quantities that represented distances that he could mark on an "open" fraction strip.

His role during the lesson seems different as well. After setting up the problem, participants worked with their partners to solve the task. When they had had time to work on the problem, the course instructor reconvened the class and asked different groups to explain their methods for finding fractions between two fractions. He offered support, asked clarifying questions, and highlighted aspects of their methods during whole class discussion. As such, he and the participants co-constructed an environment in which it was normative to explain and justify their ideas, and to represent their ideas. Interestingly, this characterization of the learning environment fits with what is commonly referred to as an *inquiry mathematics tradition* [8].

One of the earmarks of inquiry mathematics is that participants are thought to work with ideas and representations that are experientially real mathematical objects [8]. In our example, there are several instances of the instructor and the participants doing so. The instructor, for his part, often referred to the participants' ideas using the fraction strip to model ideas. As he did so, he spoke of fractions as values or as having distance. He also referred to this model as he elaborated the participants' explanations. As a result, he provided others the opportunity to

understand a group's reasoning. Further, if participants were confused, they too might imagine using the fraction strip to generate equivalent fractions. So as he facilitated the whole class discussion, he implicitly communicated that he valued these types of explanations, ones in which participants reasoned sensibly with fractions.

For their part, the participants were obliged to give explanations that were couched in their understanding about fractions. Recall, for instance, that when explaining how her group renamed 10½/110, one of the participants drew a number line to demonstrate where this fraction was located on it. She also explained that her group imagined using the fraction strip (suggested first by one of the other course instructors) to split each of the 110 pieces to find an equivalent fraction for 10½/110. Rather than simply applying a procedure for multiplying the numerator and denominator by 2, the participant essentially explained the rationale behind this procedure.

Additionally, as participants worked in small groups, they continued to hold themselves to this same standard. Ms. Smith's attempt to try Ms. Sneider's method is a case in point. As she used Ms. Sneider's method, she also had an opportunity to build some new understandings. Ms. Sneider, too, continued to pursue ideas that eventually led her to develop an argument for the density property for the Real Numbers.

Final Comments

In our discussion, we have addressed how the ideas that participants explored in the course might take on a life of their own as they worked with teachers and their students. In Ms. Smith's case, she had the opportunity not only to facilitate her students' understanding, but also to create an opportunity for Ms. Applebee to reflect on how she might facilitate students' understanding more effectively. Although we do not know if Ms. Smith capitalized on this instance, we could imagine the rich discussion that she and Ms. Applebee might have as they debriefed about this lesson. Similarly, if Ms. Sneider had the opportunity to share with the fifth grade teachers, she and her teachers could have a rich conversation about the important ideas that underpin the "funky cookie" task. Ms. Sneider, however, would need to work hard to make her instructional practices explicit to her teachers because they were not present during the pullout sessions. This said, it would be unfortunate if she did not have the opportunity to share what happened during this pullout session. Although her students might benefit from this experience, their teachers might not have the opportunity to think carefully and deeply about the nature of their students' misconceptions about fractions. Interestingly, Ms. Smith was in a much better position to positively affect her colleague's teaching practice. Although Ms. Smith was a regular

classroom teacher and Ms. Sneider was a Mathematics Specialist, in our two examples they seemed to have (temporarily) switched roles.

We have also addressed the important role that that the course *Rational Numbers* might have played in supporting the participants' mathematical learning. The instructor's role was particularly important here. He required participants to make sense of one another's methods. He also supported them as they gave explanations by asking clarifying questions and elaborating the important ideas that they addressed.

We suspect that the course experiences provided Ms. Smith and Ms. Sneider opportunities to reason deeply about fractions. We also have evidence that they drew on these ideas somehow as they made instructional decisions in order to support their students' learning. In fact, they appeared to have continued to think about ideas, even after the course had ended. As our examples illustrate, they found important ways to use their understanding of these ideas in novel, but different ways.

As we continue to explore the vast amount of data that we have gathered over the last few years, we may gain new insights into how different course experiences support the participants' daily work in schools. Perhaps we will also uncover some of the ways that the program might better serve Mathematics Specialists as they transition into their leadership roles. Can we improve on the courses that we offer? Are there other course experiences that might better support their daily work? As we traverse the data, we hope to answer these as well as other questions. At this juncture, however, we simply marvel at the extent to which the participant's work has begun to truly take on a life of its own.

Acknowledgment

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