EARLY ALGEBRA AND MATHEMATICS SPECIALISTS

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Abstract

This paper discusses early algebra as it relates to the Mathematics Specialist program. Early algebra is described based on research and readings from the body of literature focused on early algebra. Reasons why early algebra should be emphasized in elementary school mathematics are discussed, followed by a description of the role elementary school Mathematics Specialists must play if schools are to begin to focus on early algebraic instruction. Finally, some suggestions are made for ways the Mathematics Specialist program might encourage more explicitly an early algebraic approach to elementary school mathematics.

Introduction

I have been an instructor many times for the courses taught for Mathematic Specialists in Virginia. Each time I have taught one of these courses, I have deepened my own understanding of the mathematics that elementary children are capable of understanding, as well as of ways in which children come to express these understandings. However, it wasn't until I had taught all three of the courses that make up what I think of as the "Numbers" sequence (*Numbers and Operations*; *Rational Numbers and Proportional Reasoning*; and, *Patterns, Functions and Algebra*) that I began to appreciate the connectedness and complexities of these courses. Furthermore, I had not really understood how these courses work together to support a curriculum focused on early algebraic reasoning. My work with these courses has led to my interest in early algebra and the research in the field. In this paper, I want to describe a little of what is meant by early algebra, based on research and readings from the body of literature focused on early algebra. I will discuss reasons why early algebra should be emphasized in elementary school mathematics. Next, I will look at the role elementary school Mathematics Specialists must play if schools are to begin to focus on early algebraic instruction. Finally, I will make some suggestions for ways I see the Mathematics Specialist program might encourage more explicitly an early algebraic approach to elementary school mathematics.

Early Algebra: What Is It?

The *Principles and Standards for School Mathematics* describes six content standards for grades K-12 [1]. The Algebra Standard envisions students who:

- Understand patterns, relations, and functions;
- Represent and analyze mathematical situations and structures using algebraic symbols;
- Use mathematical models to represent and understand quantitative relationships; and,
- Analyze change in various contexts.

It is important to realize that this Standard spans the elementary and secondary grades. Algebra is a body of knowledge that students learn over a long span of time, beginning in the early grades. Indeed, algebra is not separate from the arithmetic studied in the elementary grades; rather, algebra and arithmetic are integrally connected.

It is also important to understand that early algebra is not what we understand as high school algebra taught in earlier grades. Most researchers echo Carpenter and Levi who claim the goal of early algebra is to develop algebraic thinking [2]. They, like other researchers in the field, conceive of algebraic reasoning as the building, expression, and justification of generalizations, representing mathematical ideas with symbols, and using those symbols to represent and solve problems [3-8]. The algebraic reasoning most appropriate for elementary school that is the focus of these researchers' work typically falls into one of two subcategories: *generalized arithmetic* and *functions.*

<u>Generalized Arithmetic</u>—This term refers to the reasoning that occurs as students recognize patterns that emerge during their study of the four basic operations, and to the claims they make and later justify, and eventually express with symbolic notation. For example, a student solving the problem 37 + 28 may take 3 from the 28 and add it to 37; the resulting problem becomes 40 + 25. At first, the student may state a generalization of what he notices as with words: "When you take an amount from one addend and add the same amount to the other addend, you still get the same total when you add them together." This serves as the basis for the symbolic expression of the relationship, $(a+b) = (a+c) + (b-c)$.

<u>Functions</u>—This term refers to the generalization of numeric patterns. Such patterns often arise from contextual situations, and may be represented with pictures, number lines, function tables,

symbolic notation, and graphs. For example, six pennies are added to a jar every day and the children analyze the growth.

An essential ingredient of early algebraic instruction is the focus on student reasoning and the discourse that allows students to identify connections among concepts, and then build on these connections to form generalizations. This discourse does not occur naturally, but rather is the result of a well articulated plan, developed by a teacher who herself understands the underlying algebraic aspects of the content. So early algebra is not just appropriate content, but also requires effective pedagogy to bring the deep meaning of the content to the surface.

Why Emphasize Algebra in Elementary Grades?

There are several reasons why an emphasis on early algebra in elementary grades is warranted. First, there is a call for early algebra on both national and state levels. Nationally, there is an emphasis on having all students complete at least one algebra course before graduating from high school. The NCTM released a position paper claiming all students should have an opportunity to learn algebra; furthermore, students need opportunities to encounter algebraic ideas across the PreK-12 curriculum [9]. Statewide, Virginia students are required by the Virginia Department of Education to pass at least three mathematics courses at or above the level of Algebra I in order to obtain a Standard Diploma [10]. The Virginia Department of Education's "Mathematics Standards of Learning" require students to explore algebraic concepts in grades K-6 [11]. Some examples of algebra content in these grades include: the formal exploration before sixth grade of the commutative, associative, and distributive properties; an understanding of equality and inequality by second grade; and, the ability to recognize and "describe a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms" by third grade [11].

Another reason to emphasize early algebra in the elementary schools focuses on issues of equity. The Equity Principle states, "All students need access each year to a coherent, challenging mathematics curriculum taught by competent and well-supported mathematics teachers" [1]. Schifter, et al. report that a focus on algebraic representations, generalizations, and connections supports students' computational fluency [6]. Furthermore, in the same article they provide evidence that working on developing algebraic reasoning supports the range of learners in a classroom. Less capable students begin to find the mathematics more accessible as they are offered more entry points; more capable students find the content associated with early algebra "challenging and stimulating." Thus, a curriculum grounded in early algebra offers greater opportunities for differentiation practices that are focused on substantial mathematical thinking.

A third argument for an emphasis on early algebra revolves around improving overall elementary mathematics curriculum. A curriculum focused on early algebra, with a constant eye on helping children build on past experiences to form generalizations that can be justified, will be much more coherent than a curriculum that "covers the Standards." A curriculum tied together by algebraic concepts makes sense, and in fact might reduce what seems to be an overwhelming amount of material to learn by providing opportunities to teach more concepts simultaneously [12]. A simple case: understanding the commutative property reduces the number of basic facts one must learn by half. A less simple case: understanding how the distributive property is applied when multiplying whole numbers allows a student to apply the same process when multiplying mixed numbers. Another less simple case: approaching fact instruction through a functional lens creates opportunity for meaningful graphing experiences, tied to pattern exploration and tabular representations.

One aspect of the work on early algebra that seems so promising is that it does not require an entire reworking of the current elementary curriculum. Rather, as Carraher, et al. state, "existing content needs to be subtly transformed to bring out its algebraic character" [7]. Kaput refers to this as "algebrafying" the elementary school curriculum [3]. This "algebrafication" requires "acknowledging the several different aspects of algebra and their roots in younger children's mathematical activity."

Enter the Mathematics Specialists

Kaput and Blanton claim "elementary teachers are in the critical path to longitudinal algebra reform, yet they typically have little experience with the rich and connected activities of generalizing and formalizing" [13]. One predictable result of this lack of experience may be a lack of depth of understanding achieved by students, even those who are successful with the *Standards of Learning*. For example, consider two students who are asked to decide if $37 + 52 > 38 + 51$. Student 1, taught by a teacher without a deep understanding of algebraic concepts, will likely resort to simply adding both sides of the equation, obtaining the same answer, and claiming the statement to be false. This is true, but an opportunity has been missed to use what Carpenter, Franke, and Levi refer to as relational thinking [14]. Also, this student has not been given an opportunity to solve this problem in ways that provide initial experiences with commutative and associative properties. Student 2, taught by a teacher with a deep understanding of the concepts and generalizations that can come from this problem, would likely solve this problem in a far different manner than Student 1. Student 2 might reason that 37 is one less than 38, but 52 is one more than 51, so the two sides are still even, using number sense and the relations between the numbers to arrive at a correct answer.

If elementary teachers lack the content and pedagogical knowledge necessary for providing the type of instruction focused on early algebraic reasoning, then clearly this is an area for their professional development. Several groups have reported their efforts in working with teachers as they begin to approach instruction of the elementary mathematics curriculum through an algebraic lens [15-17]. The approaches of these groups reflect the "algebrafication" strategy described by Blanton and Kaput [15]. This strategy is focused on classroom teacher change, approached along three avenues: 1) the "algebrafication" of instructional materials; 2) the support of students' algebraic thinking; and, 3) the creation of a classroom culture and teaching practices supportive of algebraic reasoning.

Mathematics Specialists are in a critical position to provide sustained professional development focused on algebraic reasoning. In their daily work with teachers, Mathematics Specialists regularly work with teachers to plan daily lessons and overall curriculum, work that includes modification of existing instructional resources. In schools with Mathematics Specialists, teachers are becoming better adept at listening to and exploring student reasoning, and helping students build on their own reasoning. As a result of efforts on the part of Mathematics Specialists, more and more teachers afford students opportunities to explore and deeply engage in mathematical explorations, and classroom cultures are established that respect individual reasoning. So, the basic structures of "algebrafication" are in place as a result of Mathematics Specialists in schools.

Yet for "algebrafication" to occur, early algebraic reasoning needs to become a focus of the Mathematic Specialists' work. Specialists need to provide opportunities for the teachers in their school to explore algebraic concepts for themselves in order to gain some depth of understanding of early algebra. As a Specialist works with teachers on lessons and curriculum, for example, the focus can be on underlying algebraic aspects of the concept in question, and how those aspects are brought to the forefront of discussions and developed into generalizations. Mathematics Specialists should work with teachers across the grade levels in their school to ensure that algebraic reasoning develops across concepts and from grade to grade, and that generalizations developed in one grade continue to be considered and reconceived or justified in the next. Mathematics Specialists can help teachers recognize opportunities that arise to help children form generalizations, thus supporting students' algebraic reasoning while creating a community where that reasoning is expected and valued.

Explicitly Focusing Mathematics Specialists on Early Algebra, During the Program and Beyond

Much of the work done in the "Numbers" courses of the Mathematics Specialist program focuses on algebraic reasoning. One of the first activities prospective Mathematics Specialists enrolled in the *Numbers and Operations* course engage in requires them to solve a problem like 57 + 36 using mental math. After a minute or so of reflection, participants share their strategies. Participants will propose a number of strategies, including: adding tens, then ones; changing 57 + 36 to 60 +33 or 53 +40, then completing the work with these easier, benchmark numbers; and, starting at 57 and counting on (57, 67, 77, 87, 88, 89, 90, 91, 92, 93). The language in the activity includes words like decomposing and recombining; the concepts being developed underlie the commutative and associative properties of real numbers. Other work in this course continues to examine how children use number sense to develop meaningful approaches to the four operations; these approaches often rely on (yet unstated or formulated) properties of equality.

Algebraic reasoning is an essential component of the *Rational Numbers and Proportional Reasoning* course. Work with equivalent fractions, for instance, can be viewed through a functions lens. Examples of explorations teachers encounter include looking at similar rectangles, and examining the ratio of height to width with tables and through graphing. An arrangement of nested similar rectangles on the coordinate grid reveals that the diagonals of similar rectangles fall on the same line, connecting the table to a linear function and a discussion of slope. Multiplication of fractions is analyzed through an area model, but also as the result of an operator acting on a quantity; again, a function approach.

The course *Patterns, Functions, and Algebra*, in its name and content, is the course most obviously focused on algebraic thinking. In the first half of this course, the focus is on the generalization of patterns, developing skills necessary to describe patterns with symbols. Participants develop fluency with algebraic notation as they learn how the symbols represent the physical quantities and actions. Conjectures (e.g., an odd plus an odd equals even) are justified and proven to hold over fields of numbers first with models, then symbolically. Participants use models to justify laws of equality. In the second half of the course, activities explore various functions, with an emphasis on the connections between multiple representations. Work in this course includes developing an understanding of how young children can develop an understanding of functions.

Clearly, opportunities to develop Mathematics Specialists' understanding of algebraic reasoning are available in the program courses. However, it is not clear that participants in these courses are aware of the algebraic nature of this work until they enroll in *Patterns, Functions, and Algebra.* As instructors, we miss opportunities to explicitly relate work in *Numbers and Operations* and *Rational Numbers and Proportional Reasoning* to algebra, and fail to explicitly highlight how algebra permeates the elementary curriculum. Just as a focus on early algebraic reasoning ties together the elementary curriculum, creating opportunities to teach more concepts in a connected manner and with richer understanding, a focus on algebraic reasoning could also serve to tie together "Numbers" courses in a more cohesive program.

How can the algebraic thread be made more explicit, in order to prepare Mathematics Specialists to think about early algebra in their own practice? First, some decision needs to be made as to the importance and relevance of algebraic reasoning as a unifying thread for these courses (and indeed, all content courses in the program.) If there is general agreement that algebraic reasoning should receive consistent, explicit focus, then instructional staff would benefit from professional development that highlights algebraic reasoning in the courses, and how the courses are related in this regard. This seems especially important for instructors who have not had the opportunity to teach all three of these courses, and to experience these connections themselves. The present Mathematics Specialist curriculum implicitly encourages algebraic reasoning from the onset; would it be even more powerful to encourage algebraic reasoning with more intent?

Mathematics Specialists also need support as they take on the work of implementing an early algebraic curriculum in their schools. This work should be focused on continuing to develop Mathematics Specialists' understanding of early algebra. Some of this work already occurs through conference sessions, some through local efforts. While it is not (currently) in the scope of the Mathematics Specialist program, continuing professional development focused on increasing endorsed Mathematics Specialists' knowledge of algebra could be considered in future initiatives.

Finally, a focus on algebraic instruction in elementary school is fairly new in the arena of mathematics education. Teaching number facts through a functions approach will look and feel different to teachers, administrators, parents, and children; using patterns to learn these facts is also foreign to those who see this as a rote skill. Mathematics Specialists will need to advocate for this approach, and will need support in their advocacy. Mathematics educators involved in the Mathematics Specialist program need to work with administrations to develop an understanding and support for taking this approach to the elementary mathematics curriculum, because to be effective it will require time and effort in training staff and reworking curriculum.

Early algebra and algebraic reasoning is a relatively new area of research in the mathematics education literature. There is still a lot of research that needs to be conducted to determine how children learn to reason algebraically, and what this means for instructional practices and resources. If this research is best conducted in school settings, it follows that Mathematics Specialists should play a vital role, both as research subjects and researchers. To do so, they need to be prepared.

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