

The Journal of Mathematics and Science: COLLABORATIVE EXPLORATIONS

Volume 12, 2010

PART I: SPECIAL ISSUE
What We Have Learned Symposium

PART II: REGULAR JOURNAL FEATURES

Virginia Mathematics and Science Coalition

The Journal of Mathematics and Science:

COLLABORATIVE EXPLORATIONS

Editor

P N Raychowdhury
Virginia Commonwealth University

Associate Editors

J Boyd
St. Christopher's School

J Cothron
MathScience Innovation Center

N Dávila
University of Puerto Rico

T Dick
Oregon State University

D Erchick
Ohio State University - Newark

R Farley
Virginia Commonwealth University

L Fathe
Owens Community College

B Freeouf
Washington State University

S Garfunkel
COMAP

J Garofalo
University of Virginia

T Goodman
University of Central Missouri

W Haver
Virginia Commonwealth University

W Hawkins
Mathematical Association of America

R Howard
University of Tulsa

P Keeley
Maine Mathematics and Science Alliance

S Khanna
Virginia Commonwealth University

J Lewis
University of Nebraska-Lincoln

P McNeil
Norfolk State University

R Millman
Georgia Institute of Technology

L Pitt
University of Virginia

D Royster
University of Kentucky

S Solomon
Drexel University

D Sterling
George Mason University

P Sztajn
North Carolina State University

B Williams
Williamsburg/James City Schools

S Wyckoff
Arizona State University

The Journal of Mathematics and Science: COLLABORATIVE EXPLORATIONS

Volume 12, 2010

PART I: SPECIAL ISSUE
What We Have Learned Symposium

PART II: REGULAR JOURNAL FEATURES

Virginia Mathematics and Science Coalition

INTRODUCTION

This Special Issue features articles submitted resulting from presentations made at the “What We Have Learned Symposium” held at the Roslyn Episcopal Conference and Retreat Center near Richmond, Virginia (December 1-2, 2009). The Symposium focus was on the progress of the Virginia Mathematics Specialist program to date, with special attention given to particularly successful aspects. The Symposium presentations included those by instructors and participants in the Mathematics Specialists courses which have been developed and offered in the Virginia program.

THE IMPACT OF ELEMENTARY MATHEMATICS SPECIALISTS

P.F. CAMPBELL

*Center for Mathematics Education
Dept. of Curriculum and Instruction, University of Maryland
College Park, MD 20742-1175*

N.N. MALKUS

*Dept. of Education Policy Studies, University of Maryland
College Park, MD 20742-1175*

Abstract

Elementary Mathematics Specialists are placed in schools to construct leadership roles and to provide on-site professional development addressing mathematical content and pedagogy in order to enhance instruction and to improve student achievement. A three-year, randomized, control study found that, over time, Specialists had a significant positive impact on student achievement in Grades 3, 4, and 5. This effect on student achievement was not evident at the conclusion of the Specialist's first year of placement. It emerged as knowledgeable Specialists gained experience and as the schools' instructional and administrative staffs learned and worked together. Specialists who were highly engaged with a teacher significantly impacted those teachers' beliefs about mathematics teaching and learning. In addition, teachers in schools with a Specialist were more likely to participate in a non-coaching professional activity (attending mathematics-focused grade-level meetings, observing peers' teaching, or attending schoolwide mathematics workshops). The Specialists in this study had substantial programmatic responsibilities that influenced their amount of available time for coaching teachers. Further, the Specialists in this study engaged in a high degree of professional coursework prior to and during at least their first year of placement. Findings should not be generalized to Mathematics Specialists or coaches with less expertise.

As suggested by a number of reports, many school districts have begun to define school-based positions wherein experienced and exceptional teachers serve as coaches and collegial mentors for elementary teachers, provide on-site professional development, and assume leadership roles for an elementary school's mathematics program [1, 2]. Frequently released from responsibility as a classroom teacher of record, these elementary Mathematics Specialists are charged with supporting teachers' knowledge of mathematics content and pedagogy, as well as fostering a coordinated vision of mathematics teaching, learning, and assessment in order to increase a school's instructional capacity [3]. The intent is to support collective professional habits that advance schoolwide growth and positively impact student achievement [4-6].

There is no single model defining the role of a Mathematics Specialist or coach, and a variety of exemplars are in place. One of the first references to this role was that of Joyce and Showers who used the term “peer coaching” to describe pairs of teachers who provided feedback and support to each other in an effort to advance their instruction [7]. Subsequently, Loucks-Horsley and colleagues wrote of “helping teachers” who provided mentoring to colleagues and fostered professional dialogue [8]. Regardless of the title and the distinctions in the job description, the common expectation is for an experienced practitioner who does not have administrative responsibility over teachers to advance instructional and programmatic change across a local school site by working with teachers individually and in grade-level teams.

There is a growing body of literature addressing the work and influence of Specialists or coaches, generally describing experienced challenges, intended practice, or perceptions of impact, more frequently in terms of reading and writing instruction rather than mathematics instruction [9]. However, this literature review identified only one refereed publication reporting a relationship between students’ learning of mathematics and professional development that included coaching, but this study did not control for possible prior distinctions between groups of students either by randomization or pre-testing [10].

When whole-school mathematics coaches or Specialists are placed in a school, the ultimate intent is to positively impact student learning. Yet, one could argue that an additional measure of effect is the impact of the Mathematics Specialist on teachers’ beliefs and participation in other forms of professional development addressing mathematics content and pedagogy. When elementary Mathematics Specialists work with teachers, they address the mathematical knowledge and instructional practices of teachers, but in so doing, they may also impact teachers’ beliefs and influence the degree to which teachers access other avenues for professional development. Indeed, there is evidence that teachers’ perceptions of mathematics teaching and learning change or persist in concert with their instructional practices [11, 12]. As such, teachers’ beliefs about mathematics teaching and learning and teachers’ engagement in other forms of professional development, as well as their students’ achievement, are appropriate outcomes for evaluating the effectiveness of elementary Mathematics Specialists as a vehicle for school improvement.

In 2004, the National Science Foundation (NSF) funded a collaborative project involving four universities and five school districts that collected data within a three-year, randomized, control-treatment design to investigate the work and impact of full-time Mathematics Specialists

in elementary schools in Virginia. This study's twenty-four treatment and twelve control schools represent a range of demographic and economic settings in urban, suburban, and urban-edge schools. The Specialists in this study were experienced classroom teachers who were selected by their school district and assigned to provide full-time support in a school after completing coursework in mathematics content and in leadership/coaching, as well as study of models, resources, and best practices for mathematics instruction. This article reports on the effects of these Specialists on teachers' beliefs and professional engagement, as well as on student achievement data in Grades 3-5 as measured by the high-stakes, standardized assessment administered in Virginia as required by federal *No Child Left Behind* legislation. As such, it characterizes the activity of Mathematics Specialists over time and provides insight regarding coaching as a vehicle for instructional reform. This article addresses the following research questions:

- What activities did elementary Mathematics Specialists engage in and what proportion of their total time did they spend completing those differing duties?
- What was the impact of elementary Mathematics Specialists on student achievement as measured by Virginia's high-stakes standardized assessment?
- What was the impact of elementary Mathematics Specialists on teachers' beliefs about mathematics teaching and learning, as influenced Specialists' degree of involvement with individual teachers?
- What was the impact of elementary Mathematics Specialists on teachers' involvement in other forms of professional development?

Method

Specialists—Five school districts in Virginia, representing urban, urban-edge, and rural-fringe communities, identified one or more triples of schools with comparable student demographics and comparable traditions of student performance on state mathematics assessments. Triples of schools, rather than pairs, were identified in order to yield comparable school placement sites for two differing cohorts of Mathematics Specialists while maintaining corresponding control sites. This study accessed Specialists who were participating in a NSF-funded teacher enhancement effort that required two cohorts of Specialists in order to develop and refine the mathematics content, pedagogy, and leadership courses completed by the Specialists.

One school was randomly selected from each of the 12 triples by the first author and was assigned a Mathematics Specialist by a cooperating school district during the 2005-06 school

year. A cohort of twelve Specialists completed five mathematics content courses and one leadership/coaching course during 2004 and 2005 prior to placement, as well as a second leadership/coaching course during their first year of service as an elementary Mathematics Specialist. Of the twelve Specialists in this first cohort, eleven remained in one of the original treatment schools for three school years (August 2005–June 2008). One treatment school closed due to redistricting after the 2006-07 school year, and one Specialist in this cohort retired at that time, accepting a position as half-time supervisor of Specialists across that school district. The Specialist displaced by the school closing was reassigned to the school formerly supported by the newly retired Specialist, thus maintaining placement of a third-year elementary Mathematics Specialist across all of the remaining Cohort 1 schools during Year 3 of the study.

A second cohort of twelve Specialists completed a similar offering of these same content and leadership courses during 2006 and 2007. The first author randomly selected one of the two remaining control schools in each of the original triples of schools; these sites were identified as the Cohort 2 schools. Cooperating school districts then assigned each of the twelve Specialists in the second cohort to one of the Cohort 2 schools for two school years (August 2007– June 2009).

School districts were paid an allotment of \$25,000 per coach, per year in order to offset the cost of replacement classroom teachers. Specialists were also paid an annual stipend of \$2,500 for participating in the data collection phase of the study. All twenty-four Specialists were female. Eight of the Specialists are African-American; one coach is Asian; the remaining Specialists are White.

Teachers—Over the course of four years, there were 1,769 teachers of K-5 mathematics in the thirty-six cooperating schools who participated in the study. The teachers in the three cohorts of schools did not differ substantively in terms of their professional experience or demographics (see Table 1).

Table 1
Grade K-5 Teachers' Professional Demographics (2005-09)

Year	Cohort 1 ^a			Control 1/Cohort 2				Control Throughout			
	05-06	06-07	07-08	05-06	06-07	07-08	08-09	05-06	06-07	07-08	08-09
Master's Degree (%)	36.4	38.2	35	40.5	46.7	45.9	46.1	35.1	40.6	41.8	42.0
Years of Teaching Experience (%)											
1 or 2 years	16.3	18.7	17.9	24.8	15.6	14.8	5.7	14.5	11.3	13.1	6.2
3 or 4 years	10.1	9.8	6.8	11.8	18.0	17.2	12.7	8.4	14.3	14.8	14.4
5 through 9 years	34.9	32.5	33.3	27.0	28.7	29.5	29.6	19.8	21.1	16.4	20.9
10 or more years	38.8	39.0	41.9	35.9	37.7	38.5	46.2	57.3	53.4	55.7	53.9
Certified Teachers (%)	98.4	95.9	96.6	94.1	94.3	95.9	95.8	93.9	94	96.7	96.1
Female (%)	93.8	93.5	92.3	83.7	86.1	83.6	85.6	87.8	86.5	88.5	87.6
Race/Ethnicity (%)											
American Indian/ Alaskan Native	0	0	0	0.7	0.8	0.8	2.2	0	0	0.8	1.0
Black/African- American	25.6	22.0	20.5	30.7	30.3	27.0	27.2	23.7	27.1	27.9	34.0
White	72.1	74.0	76.1	65.4	68.0	70.5	66.7	73.3	68.4	64.8	60.5
Asian/Asian American	0	0	0	1.3	0	0	0	1.5	2.3	1.6	0.7
Hispanic/Latino	1.6	2.4	1.7	0.7	0.8	0.8	1.6	0	0	0	0.7
More than one race	.8	1.6	1.7	0.7	0	0	1.0	0.8	1.5	3.3	2.3
<i>n</i> of Teachers	333	316	309	393	333	332	314	356	323	319	306

^a 12 schools in 2005-07; 11 schools in 2007-08

Data Sources: Coaches' Activity and Engagement with Teachers—To account for their changing actions in school, coaches detailed the nature and duration of their daily activities using a data collection-transmittal program operating on a Personal Digital Assistant (PDA; Dell Axim X50™). The Instructional Specialist Activity Manager (ISAM) is a menu-oriented, entry interface that allows coaches to log the duration and category of their daily activity and to log a weekly reflection describing their level of engagement with particular teachers in a given week, with teacher identification cycling over the course of the school year.

Within the Daily Activity Log option of ISAM, coaches chronologically indicate the duration of an activity and then “click” the primary identification of that activity. Based on a branching network, activities of interest trigger the presentation of more detailed sub-choices which coaches again select by “clicking” on the button of interest. After entering the activity for a time period, the coaches may review and, if necessary, modify their entry. After the activities of a complete day are entered, coaches may review the day’s entries and, if necessary, modify the listing prior to confirmation.

The Weekly Reflection Log lists the names of 1/6 of the teachers in a school each week, with the names cycling over a six-week period so that each teacher’s name appears once in that period and six times over the course of a school year. This log asks the coaches to reflect on their interaction with a named teacher and indicate the teacher’s level of engagement with the coach during group-directed, grade-level planning sessions over the past month and during individual interaction over the past ten days. As with the Daily Activity Log, coaches may review and modify their entries prior to confirmation.

Daily and Weekly confirmed data were subsequently transmitted over the Internet onto a comprehensive data management platform. This platform was housed on a server at the authors’ university.

Data Sources: Student Mathematics Achievement Data—All students in Grades 3 through 8 in Virginia are expected to complete a statewide, standardized achievement test in mathematics termed the *Standards of Learning Assessment (SOL)* annually. Through administration of this high-stakes measure and the aligned collection of student demographic data, Virginia meets the expectations for assessment as required under the federal *No Child Left Behind* legislation. Data from the *SOL* include a total scale score for mathematics (possible scores ranging from 200 to 600) addressing content spanning: number and number sense; computation and estimation; measurement and geometry; probability and statistics; and, patterns, functions, and algebra. The *SOL* are administered annually, typically during the last half of the month of May.

While the *SOL* in Grades 3 and 5 have been administered since the 2001-02 school year, the Grade 4 *SOL* were administered for the first time during the 2005-06 school year, the first year of placement of Specialists in this study. Further, while the Grade 4 and Grade 5 *SOL* only assess content associated within the grade-level standards of that single grade, the Grade 3 *SOL* assessment measures content from Kindergarten through Grade 3. Thus, the analysis that follows

separately considers the third-, fourth-, and fifth-grade students' scores across three years (2005-08).

For each grade level, the primary dependent variable was the overall *SOL* Mathematics scale score across three years. This dependent variable posed two challenges for these analyses. First, in each grade level, the distribution of test scores shifted in a non-linear fashion as the difficulty of the *SOL* Mathematics assessment varied from year to year. Second, while the range for the *SOL* scale scores was 200-600, there was a substantial but varying number of students in each year and in each grade achieving a score of 600. This ceiling effect was problematic because it increased the type II error rate, making it more difficult to detect true differences between groups. Because of these two challenges, it was not possible to standardize scores across years. Therefore, in order to control for differences in the testing year, this analysis used the scale scores in the original metric and included binary indicators for each testing year.

Ideally, an analysis examining impact of a treatment on student achievement would include student-level prior achievement in the model. However, this was also problematic, in part because of the ceiling effect and the inability to standardize scores. Further, there were no prior-year *SOL* scores for Grade 3 students in any school year or for Grade 5 students in the 2005-06 school year. In addition, when prior-year *SOL* assessments were administered, missing data due to student mobility was present and not evenly distributed across schools as coded by Title I status and minority composition.

Because controlling for student-level prior achievement was not possible, two school-level measures of the prior academic tradition were included to control for differences between schools at the extremes of the prior student achievement distribution. Low Academic Tradition and High Academic Tradition identified those schools whose mean 2004-05 *SOL* Mathematics scale scores in both Grades 3 and 5 were at least one standard deviation below or one standard deviation above the 2004-05 sample mean for all thirty-six schools.

Data Sources: Teacher Beliefs Data—The teachers of mathematics in the control and treatment schools completed a beliefs survey in Fall 2005, and again in Spring 2006, 2007, 2008, and 2009. This assessment was constructed using a 20-item instrument developed by Ross and colleagues with the addition of ten additional items addressing equity and directed instruction [12]. Using the 5-point Likert scale, respondents rated each of thirty statements on a scale of 1 (“strongly disagree”) to 5 (“strongly agree”). The statements in the survey reflected perspectives about

mathematics curriculum and instruction, and perspectives regarding the needs of students and student understanding. Factor analysis identified two orthogonal factors: items that distinguished beliefs emphasizing directed teaching and mathematical structure as a basis for curriculum (Traditional) and items reflecting a perspective emphasizing the development of students' principled knowledge and supporting student efforts to "make sense" of the mathematics (Making Sense). Table 2 presents illustrative items from the beliefs survey. The reliability of the total 30-item scale as indicated by Cronbach's alpha is .797.

Table 2
Exemplar Items from the Beliefs Survey

<p>Items Reflecting a Traditional Perspective</p> <p style="padding-left: 40px;">Learning mathematics requires a good memory because you must remember how to carry out procedures and, when solving an application problem, you have to remember which procedure to use.</p> <p style="padding-left: 40px;">The best way to teach students to solve mathematics problems is to model how to solve one kind of problem at a time.</p>
<p>Items Reflecting a "Making Sense" Perspective</p> <p style="padding-left: 40px;">Students can figure out how to solve many mathematics problems without being told what to do.</p> <p style="padding-left: 40px;">I don't necessarily answer students' math questions, but rather let them puzzle things out for themselves.</p>

Data Sources: Teacher Professional Engagement Data—Each spring, teachers completed a survey addressing the nature and degree of their involvement in professional development over the past school year. These entries described professional development opportunities offered within and outside of schools and the amount of time spent in those activities.

Analysis and Results

Activity of the Elementary Mathematics Specialists—The ISAM Daily Activity Logs from the PDA's characterize the duration and nature of Specialists' activity across up to three years of placement in a school. Although Specialists were not expected to complete tasks related to their work responsibilities outside of their contract day, many Specialists did so. Therefore, this presentation of the activity data distinguishes between those two contexts for activity.

Table 3 presents the proportional distribution (percent) of Specialists' mean time over activity within the contracted workday for Cohort 1 and Cohort 2 Specialists. If a Specialist was absent from work on a contract day, that time is not reflected in Table 3. The mean length of a contract day for a Specialist was 7 hours, 22 minutes; the median length of a contract day for Specialists was 7.5 hours. Thus, on average, the Specialists were paid to spend 36 hours, 50 minutes at school each week with a forty-week school calendar. In terms of hours per day, the values in Table 3 may be interpolated according to the formula that 13.6% is equivalent to 5 hours per week (comparable to 1 hour per day) and 2.7% is equivalent to 1 hour per week.

Table 3
Percent of Mean Contract-Day Time over Specialist Activities by Cohort and Year

Activity	Cohort 1 2005-06 (Year 1)	Cohort 1 2006-07 (Year 2)	Cohort 1 2007-08 (Year 3)	Cohort 2 2007-08 (Year 1)
Coaching Teachers (Individual Teachers and Grade-Level Teams)	21.9	13.1	12.9	10.2
Preparing for Teaching/Coaching	11.8	12.4	12.5	11.8
Supporting Assessment	10.6	13.5	13.7	12.5
Teaching or Supporting Students (Not Demonstration or Co-Teaching)	3.0	4.4	4.5	3.6
Supporting the School Mathematics Program	5.0	4.2	5.1	5.1
Performing School-Based Duties	6.5	9.2	10.4	9.8
Materials Management/ Communication Tasks	9.7	11.0	11.8	11.4
Attending Meetings	9.2	6.8	6.7	9.5
Engaging in Personal Professional Activity	13.2	14.7	10.9	14.4
Non-Educational Activities (lunch, travel, all-school event)	9.0	10.8	11.3	11.8

The amount of contract-day time that the Cohort 1 Specialist spent coaching individual teachers decreased over the three years. The amount of time that each of the two cohorts spent coaching teachers was more consistent when the year of work was constant (both cohorts in 2007-08) than when the extent of experience was constant (Cohort 2 in 2007-08 and Cohort 1 in 2005-06). This may mean that during 2007-08 there were common outside influences impacting

Specialists' decisions as to how much available time they had to spend working with individual teachers.

Over that same time period, the amount of time that Cohort 1 Specialists spent addressing assessment increased. The Cohort 2 Specialists spent somewhat less time than the more experienced Cohort 1 Specialists addressing assessment during 2007-08, primarily because Cohort 2 Specialists had less time devoted to developing assessments and to assessment management. This may reflect the increased managerial expertise presumed of Cohort 1 Specialists during their third year of placement. The increase in Cohort 1 assessment activity and the frequency of Specialist time devoted to assessment responsibilities across the two cohorts was evident in each of the five school districts, with frequency of assessment time being a consistently modal feature of the urban districts. Because Mathematics Specialists were not assigned across all schools in a district, shifting of Specialists' time to assessment responsibilities is probably a local school response to concerns associated with assessment demands, a response that is evident within and between districts. In contrast, the increase in time Cohort 1 Specialists spent teaching or supporting students without an observing teacher present (thus not coaching through demonstration teaching, modeling, or co-teaching) varied by individual Specialists, not districts. Therefore, this was most likely a reflection of a principal's request and not a consistent response to district policy or pressure.

The time Specialists spent in meetings that did have a mathematics focus was quite consistent within districts, while being unique across districts. This indicates that a Specialist's attendance at a meeting addressing mathematics was likely not an individual decision, but reflects an expectation of either a principal or district office. As Cohort 1 Specialists gained expertise, local administrators were less likely to expect their attendance at a meeting when the agenda was not related to mathematics.

The prevalence of activity associated with personal professional development reflects the fact that all Specialists completed the second leadership/coaching course during their first year of placement. Further, many of the Specialists in each cohort completed an additional graduate course or two during their first year of placement as they completed requirements for a master's degree within the following summer or fall semester.

The amount of time that Specialists spent addressing communication, such as e-mail correspondence, was more comparable by academic year than by year of expertise. All of the

participating school districts provide e-mail addresses and access to their instructional and administrative staffs. The increase in time evidenced between 2005-06 and 2007-08 is most likely a reflection of changes in school culture. In contrast, the prevalence of school-based duties is most likely a project-related artifact. The Specialists advised each other to “volunteer for bus duty” as a way to build trust and entrée into their school placements, noting that this was a time when few, if any, teachers would be available to meet with a Specialist.

The Specialists varied in terms of how much out-of-school time they devoted to responsibilities associated with their work. On average, the Specialists spent 4.5 hours a week completing work-related activity for which they were not paid. This is equivalent to approximately 180 hours or twenty-four extra contract days per year.

Table 4 presents the Specialists’ mean out-of-school time in hours per category of activity by school year. A substantial portion of this out-of-school time was allotted to personal professional activities. Much of this time was related to the graduate coursework that a number of Specialists were completing during their first two years of placement, as this pattern diminished markedly in the third year of Cohort 1 data entries after the degree-seeking Specialists in that cohort had completed their degrees. Materials management and communication also demanded much of the Specialists’ time outside of the contracted workday. Over half of this time was spent attending to communication tasks involving e-mail, telephone calls, or the production of flyers while the remaining time was split between PDA data entry and activity associated with supporting the purchasing, distribution, and management of educational materials.

Table 4
Specialists' Mean Out-of-School Time in Hours per Activity Category by Year

Activity	Cohort 1 2005-06 (Year 1)	Cohort 1 2006-07 (Year 2)	Cohort 1 2007-08 (Year 3)	Cohort 2 2007-08 (Year 1)
Coaching Teachers	9.69	8.24	15.8	22.1
Preparing for Teaching/Coaching	31.4	26.5	18.68	11.13
Supporting Assessment	13.53	12.72	10.77	6.97
Teaching or Supporting Students (not Demonstration or Co-teaching)	4.68	4.02	3.17	1.97
Supporting the School Mathematics Program	17.05	11.28	8.17	5.97
Performing School-Based Duties	26.04	25.24	13.57	13.88
Materials Management/Communication Tasks	36.84	21.14	22.51	29.60
Attending Meetings	25.83	14.86	14.3	14.28
Engaging in Personal Professional Activity	55.53	43.98	19.4	30.02
Non-Educational Activities within Out-of-School Work Time (lunch, break, travel)	27.86	16.84	10.5	17.35
Mean Total Hours per Year	248.45	184.82	136.87	153.27

The Cohort 1 Specialists were more likely to spend their out-of-school work time preparing for coaching or teaching (demonstration teaching, modeling, or co-teaching) rather than coaching teachers, but the Cohort 2 Specialists evidenced the reverse pattern. For both cohorts, coaching time outside of the contract day was typically spent working with individual teachers, rather than with groups of teachers or grade-level teams. The out-of-school time devoted to the performance of duties almost exclusively involved monitoring students, while outside-of-contract-day work associated with the school mathematics program involved activities such as a school's annual Family Math Night.

Mathematics Achievement—In order to determine whether elementary Mathematics Specialists impacted student mathematics achievement as measured by standardized state assessments, this

analysis accessed data from 24,749 student *SOL* scale scores drawn from Grades 3, 4, and 5 of thirty-six treatment and control schools over three years. Hierarchical linear modeling (HLM) was used to analyze the data. Across these three years, this sample included 1,169 teachers/classrooms of students in Grades 3, 4, and 5, of which 368 were in Cohort 1 schools, 406 were in Control 1/Cohort 2 schools, and 395 were in control schools throughout. Two analyses were conducted. The Treatment versus Control analysis compared three years of mathematics achievement scores of students in the control schools to scores of students in the treatment schools, noting whether the achievement scores of students in the treatment schools were from the three years of data from the Cohort 1 schools (2005-08) or from the one year of data from the Cohort 2 schools as collected during the third year of the study (2007-08; Cohort 2 Year 3). A second Cohort-by-Year versus Control analysis compared three years of mathematics achievement scores of students in the control schools to scores of students in the treatment schools, noting whether the achievement scores of students in the treatment schools were from the first (Cohort 1 Year 1), second (Cohort 1 Year 2), or third (Cohort 1 Year 3) year of Specialist placement in a Cohort 1 school or from the first year of Specialist placement in a Cohort 2 school (Cohort 2 Year 3) during the third year of the study.

Both the Treatment versus Control analysis and the Cohort-by-Year versus Control analysis entered identical student-level and classroom-level variables in their respective statistical models. In particular, for each grade, the analysis included controls for a student's age at the time of testing (*AgeTest*), as well as binary indicators to provide controls for student gender (*Female*), student Limited English Proficiency status (*LEP*), student special education status (*SpecEd*), student free- and/or reduced-meal status (*FARM*), and student minority status (*Minority*). The reference categories for these binary indicators were male gender and not accessing special services. The models also included two indicators to identify whether the *SOL* tests were being administered in the second or third year of the study (*2007 Test* and *2008 Test*, respectively). The reference category for the year of the test was the first year of the study, the 2006 Test administered at the end of the 2005-06 school year. The only teacher/classroom-level variables that were significant or improved model fit were a binary indicator for teachers with master's degrees (*Masters*) and measures indicating years of teaching experience (*1-2 Years Experience*, *3-4 Years Experience*, or *10+ Years Experience*). The reference category for teacher experience was 5-9 years of teaching experience.

Additional school-level variables provided controls for schoolwide Title I services (Title I) and a standardized school size measure (School Size). An additional variable is the academic tradition of the school (Low Academic Tradition and High Academic Tradition).

Findings from the Treatment versus Control analysis are presented in Table 5, with the statistics for differing independent variables presented in each row and the grouped columns specifying the grade. In all three grades, the Cohort 1 coefficients were positive and significant indicating the positive impact of elementary Mathematics Specialists on student achievement. In Grade 3, students in Cohort 1 schools averaged 10.7 points, or 14% of the Grade 3 pooled standard deviation (SD) higher than the mean on the *SOL* Mathematics scaled score ($p = 0.040$). In Grades 4 and 5, students in Cohort 1 schools scored 13.7 ($p = 0.0095$) and 15.3 ($p = 0.004$) points above the mean, respectively, which corresponds to 18% SD on the Grade 4 tests and 19% SD on the Grade 5 tests. In contrast, the Cohort 2 Year 3 variable, representing the placement of a first-year coach during the third year of the study, was not significant in any of the grade-specific analyses.

Table 5
Parameter Estimates and Standard Errors for Specialist Effects on
Student SOL Mathematics Overall Scale Score by Grade

Scale Score	Grade 3			Grade 4			Grade 5		
	Coefficient		SE	Coefficient		SE	Coefficient		SE
Intercept	493.95	***	2.34	470.63	***	2.45	496.95	***	2.46
Student Variables									
Age at Test	-8.49	***	1.61	-14.37	***	1.43	-13.29	***	1.56
Female	-1.86		1.26	-7.72	***	1.55	-0.69		1.64
LEP	-8.02		7.25	-18.71	**	6.33	-21.89	**	6.74
Special Education	-41.84	***†‡	3.45	-40.17	***†‡	3.30	-51.53	***†‡	4.36
Free or Reduced									
Meal	-17.10	***	2.20	-17.80	***†	2.41	-18.24	***†	2.78
Minority	-35.76	***	2.16	-33.74	***	2.21	-27.68	***	2.08
2007 Test	3.71		5.26	18.52	**	5.20	20.32	***	5.37
2008 Test	7.32		5.96	12.12		7.31	11.30		6.09
Teacher Variables									
Master's Degree	2.12		2.42	0.21		2.94	-5.14		3.39
1-2 years Experience	-7.87		5.15	-4.96		5.24	-13.67	*	5.98
3-4 Years Experience	-6.54		4.27	-7.39		5.02	-6.01		5.38
10+ Years Experience	0.76		2.95	10.45	*	4.11	10.60	**	3.75
School Variables									
Title I School	7.04		5.57	4.70		5.67	12.99	*	5.34
High Academic									
Tradition	35.53	***	8.97	39.51	***	9.71	51.45	***	6.47
Low Academic									
Tradition	-13.26		7.65	-17.06	*	7.88	-2.08		9.65
School Size	-2.78		2.48	-8.43	**	2.69	-7.98	**	2.44
Cohort 1	10.71	*	5.05	13.68	**	5.17	15.25	**	5.08
Cohort 2 Year 3	-3.89		7.43	9.08		8.31	-3.16		8.74
Variance Estimates									
	Variance		χ^2	Variance		χ^2	Variance		χ^2
Student-level	3927.03		...	3776.08		...	4319.54		...
variance (σ_2)									
Class-level	382.88	***	676.93	523.16	***	518.86	554.62	***	569.58
variance ($\tau_{00\pi}$)									
School-level	307.98	***	288.3	277.73	***	238.65	289.61	***	207.43
variance ($\tau_{00\theta}$)									

* $p < .05$ ** $p < .01$ *** $p < .001$ † Un-modeled level 2 random effect ‡ Un-modeled level 3 random effect

At the classroom level, students whose teachers had a master's degree did not have significantly different *SOL* scores than students taught by teachers without a graduate degree. The effects of teacher experience were not consistently significant across the grade-level analyses, but were in the expected direction with students with early-career teachers having somewhat lower *SOL* scores than did students of teachers with 5-9 years of teaching experience. The magnitude and significance of student achievement differences associated with teacher experience generally increased by grade.

Across all three grades, the individual effects of age, poverty, race/ethnicity, and special education status had consistently significant negative effects on total *SOL* mathematics scores ($p < 0.001$). The effects of gender and LEP status were negative, but not consistently significant. In 2007, the average *SOL* Mathematics scale score was significantly higher for Grades 4 and 5 (25% SD, $p < 0.01$). In 2008, the year-of-test effects were not significant; however, the magnitude of the coefficients underscores the importance of including these controls in the model.

The Treatment versus Control analysis did not identify a significant effect in the Cohort 2 Year 3 variable, while there was a significant positive effect associated with the Cohort 1 Specialists over the three years. The Cohort-by-Year versus Control analysis permitted an examination of whether this difference in findings reflected the differing amounts of time the elementary Mathematics Specialists in the two cohorts had to work with teachers and the school mathematics program or whether this difference reflected a cohort effect. Grade-specific findings from these analyses are reported in Table 6.

Table 6
Parameter Estimates and Standard Errors for Specialist by Year Effects on Student SOL Mathematics Overall Scale Score by Grade

Scale Score	Grade 3			Grade 4			Grade 5		
	Coefficient		SE	Coefficient		SE	Coefficient		SE
Intercept	493.91	***	2.31	470.66	***	2.50	497.05	***	2.44
Student Variables									
Age at Test	-8.50	***	1.61	-14.37	***	1.43	-13.28	***	1.56
Female	-1.86		1.26	-7.72	***	1.55	-0.71		1.64
LEP	-8.00		7.25	-18.72	**	6.33	-21.85	**	6.71
Special Education Free or Reduced Meal	-41.82	***†‡	3.45	-40.17	***†‡	3.30	-51.62	***†‡	4.36
Minority	-35.76	***	2.16	-33.73	***	2.21	-27.64	***	2.09
2007 Test	2.69		6.91	17.48	**	6.56	16.21	*	7.13
2008 Test	3.41		7.45	11.87		11.22	6.17		8.13
Teacher Variables									
Master's Degree	2.15		2.43	0.21		2.94	-5.03		3.37
1-2 Years Experience	-7.67		5.09	-5.00		5.27	-13.15	*	6.02
3-4 Years Experience	-6.56		4.27	-7.41		5.01	-5.98		5.31
10+ Years Experience	0.82		2.95	10.45	*	4.10	10.66	**	3.73
School Variables									
Title I School	7.23		5.71	4.57		5.66	13.49	*	5.42
High Academic Tradition	35.44	***	8.82	39.46	***	9.65	51.80	***	6.10
Low Academic Tradition	-13.88		7.89	-17.03	*	7.88	-2.77		9.53
School Size	-2.81		2.49	-8.45	**	2.70	-8.09	**	2.36
Cohort 1 Year 1	6.81		5.83	12.27		7.26	6.34		7.67
Cohort 1 Year 2	10.38		8.83	15.35	*	7.62	19.61	*	7.82
Cohort 1 Year 3	16.48		11.03	13.25		11.98	20.31	*	9.24
Cohort 2 Year 3	-1.11		8.32	8.86		10.90	-0.63		9.44
Variance Estimates									
Student-level variance (σ_2)	3926.95		...	3776.06		...	4319.37		...
Class-level variance ($\tau_{00\pi}$)	382.60	***	677.02	523.33	***	518.83	554.37	***	569.50
School-level variance ($\tau_{00\beta}$)	305.20	***	287.61	276.90	***	238.26	276.01	***	203.25

* $p < .05$ ** $p < .01$ *** $p < .001$ †Un-modeled level 2 random effect ‡Un-modeled level 3 random effect

A comparison of Tables 5 and 6 reveals no substantive changes in any student and classroom coefficients, except for the year-of-test control variables (2007 Test; 2008 Test). With Cohort 1-by-Year in the model, the differences between Cohort 1 and Control students' mean scale scores each year is removed from the 2007 Test and 2008 Test estimates and attributed to the year-specific Cohort 1 estimate. Thus, the 2007 Test and 2008 Test coefficients are reduced

as the Cohort 1 coefficients in those years increase. This pattern is particularly evident in the Grade 4 analysis.

The Cohort 1-by-Year variables reported in Table 6 reveal a consistent pattern of results over time although, as expected, the increased variance of the estimates reduced the number of significant coefficients. In Grade 3, none of the Cohort 1-by-Year variables were significant. In the first year of the study, the *SOL* mathematics scores of the Cohort 1 students were, on average, 6.8 points (9% SD, $p = 0.25$) higher than those of the students in the control schools. In Year 2, the coefficient increases to 10.4 points (14% SD, $p = 0.24$), and in Year 3 it increases to 16.5 points (22% SD, $p = 0.14$). While increasing coefficients are apparent in the second and third years of placement of an elementary mathematics coach, the increasing Cohort 1-by-Year coefficients in this analysis are not significant, due in large part to the increased standard errors associated with this more conservative analysis.

In Grade 4, there is a similar pattern. In the first year on average, the Cohort 1 students scored 12.3 points higher (17% SD, $p = 0.09$) than the control students on the *SOL* Mathematics assessment, though this coefficient is not significant. In the second year, the coefficient was significant as, on average, Cohort 1 students scored 15.4 points higher (21% SD, $p = 0.046$) than Grade 4 students in the control schools. In the third year of the study, the coefficient for Cohort 1 students fell somewhat to 13.3 points (18% SD, $p = 0.27$), with a substantially larger standard error.

In the Grade 5 analysis, the pattern of growth is more compelling with larger and significant differences in both the 2007 and 2008 testing years. The Cohort 1 Year 1 coefficient for Grade 5 was small and non-significant at 6.3 points (8% SD, $p = 0.41$). However, during the second and third year of the placement of a coach, on average the Cohort 1 students scored 19.6 (25% SD, $p = 0.01$) and 20.3 (25% SD, $p = 0.03$) points higher respectively, than the students in the control group. Both of these estimates are statistically significant.

Across all three grades, the Cohort 2 Year 3 variable had smaller coefficients in the Cohort-by-Year versus Control analysis as compared to the coefficients in the Treatment versus Control analysis over three years. The reductions in these coefficients were due to the entry of the Cohort 1-by-Year variables. These more accurate Cohort 2 Year 3 estimates are consistent with the pattern of Cohort 1 coefficients, with no statistically significant improvements in student scores in the first year of coaching, with larger increases evident in following years.

Teacher Beliefs—In order to determine whether elementary Mathematics Specialists impacted

teachers' beliefs about mathematics teaching and learning, HLM analyses examined data from teacher beliefs surveys as collected from K-5 teachers in thirty-six treatment and control schools over four years. Across these years, this included 906 surveys from teachers in Cohort 1 schools (2005-08), 1,264 surveys from teachers in Cohort 2 schools (Control status during 2005-07; Specialist present during 2007-09), and 1,198 surveys from teachers in Control schools throughout 2005-09. Analyses were conducted on data drawn from items associated with the Traditional factor and on data drawn from items associated with the Making Sense factor.

Parallel HLM school-level and classroom-level models were applied in each analysis of beliefs data. Two school-level variables identified whether the school had an elementary Mathematics Specialist (Treatment School), as well as whether the school was a Title I school (40% or more of the enrolled students eligible for Title I services). There were five teacher-level variables: namely, a grand-mean-centered continuous variable noting years of teaching experience (Teacher Experience); gender (Female); a baseline measure of teacher beliefs on each factor as collected when a teacher entered the study (Prior Making Sense Beliefs and Prior Traditional Beliefs); and, whether a teacher was highly engaged with an elementary Mathematics Specialist (Highly Engaged Teacher). The inclusion of the variable measuring baseline teacher beliefs ensured that differences in teacher beliefs reflected changes during the time under study, rather than absolute differences in teacher beliefs that were present prior to the study and remained throughout.

While the elementary Mathematics Specialists were responsible for working with an entire school, in practice their level of engagement with teachers varied. The Weekly Reflection Logs on the PDA's provided a categorical estimation of the quantity of both individual and group-level interaction between teachers and Specialists. These reflections were entered approximately six times per year. In order to yield an annual measure of high engagement between a Specialist and a teacher, these entries were coded according to a 0/1 score (see Table 7) and summed. The proportion of these summed values to possible points (1 point per entry) yielded an Engagement rating. This proportion was then translated to a 0/1 binary measure (High Engagement) to indicate if there was or was not a high level of engagement between a teacher and a Specialist. Teachers who had an Engagement rating of 0.75 or higher were coded as 1 on the binary indicator of Highly Engaged Teacher.

Table 7
0/1 Engagement Values for ISAM Weekly Reflection Logs

	Engagement Value
Individual Engagement	
Teacher seeks the Specialist	1
Teacher is a professional colleague to the Specialist	1
Teacher supports other teachers	1
Teacher accepts the Specialist	0
Teacher avoids the Specialist	0
Teacher absent from school over past 10 days	0
Engagement During Group Planning/Team Meetings	
Teacher fully participates	1
Teacher organizes colleagues	1
Teacher contributes only when asked	0
Teacher decided not to attend a meeting	0
Teacher passively attends a meeting	0
No planning or group meeting scheduled over past 10 days	0
Teacher was not available to attend a meeting	0

These models explained all of the between-school variation and 41% of the individual variation on the Traditional beliefs factor and 37% of the variation between individuals on the Making Sense factor.

As reported in Table 8, the most powerful teacher-level predictor of teacher perspectives on the Making Sense factor was a teacher's baseline Making Sense factor score. Teachers whose baseline Making Sense scores were 1 SD higher than the mean at the initiation of the study had, on average, 59.3% higher scores on the final Making Sense measure. The indicator for Title I was significant and negative, indicating teachers in Title I schools actually had a moderately lower Making Sense factor score by about 15% SD (-21.2% + 6.2%). While, on average, teaching experience did not significantly impact Making Sense factor scores (an increase of 0.2% per year of teaching experience), each year of teaching experience in Title I schools was associated with an additional 0.6% SD decrease in the Making Sense factor score per year of experience. Beliefs of teachers in schools with an elementary school Mathematics Specialist did

not differ significantly in terms of the Making Sense perspective from those teachers in the Control schools, unless the teacher was highly engaged with the Mathematics Specialist. Teachers who were highly engaged with their Mathematics Specialist had, on average, a statistically significant 12.5% SD higher measure on the Making Sense beliefs scale.

Table 8
Effects of Elementary Mathematics Specialists on Teacher Beliefs
Reflecting a Making Sense Perspective

	Coefficient	Standard Error	
Intercept	.062	.014	***
Treatment School	.009	.031	
Title I School	-.212	.028	***
Teaching Experience	.002	.001	
Title I School	-.006	.003	*
Female	.015	.004	***
Prior Making Sense (Baseline)	.593	.014	***
Teacher Highly Engaged with an Elementary Mathematics Specialist	.125	.037	***
Variance Estimates	Variance		χ^2
Reliability (λ)	.883		
Teacher Level Variance (σ^2)	.914		
School Level Variance (τ_{00})	.081		95.61
ICC (ρ)	.081		
Final Teacher Level Variance (σ^2)	.575		
Teacher Variance Explained	37.1%		

* $p < .05$ ** $p < .01$ *** $p < .001$

Note: Schools' Title I status explained enough of the differences between schools on the intercept that there were no longer significant differences between schools. Thus, this model has no random effects and coefficients are OLS estimates.

In the analysis of the Traditional beliefs data, the baseline measure of Traditional beliefs was the most powerful predictor in the model indicating that a standard deviation increase in the

initial measure of Traditional beliefs was associated with a 67.5% SD increase in the final measure (see Table 9). Teachers who taught in Title I schools were substantively more likely to have Traditional beliefs by 33.5% SD (-3.7% + 37.2%). While, on average, teaching experience did not significantly impact Traditional factor scores (a decrease of 0.2% per year of teaching experience), each year of teaching experience in Title I schools was associated with a significant additional 0.6% SD increase in the Traditional factor score per year of experience. These results indicate that teachers in Title I schools held much more traditional beliefs than their counterparts in non-Title I schools and these differences became more pronounced with increasing years of teaching experience.

Table 9
Effects of Elementary Mathematics Specialists on Teacher Beliefs
Reflecting a Traditional Perspective

	Coefficient	Standard Error	
Intercept	-.037	.012	**
Treatment School	-.029	.027	
Title I School	.372	.028	***
Teaching Experience	-.002	.001	
Title I School	.006	.002	*
Female	-.011	.003	**
Prior Traditional (Baseline)	.675	.014	***
Teacher Highly Engaged with an Elementary Mathematics Specialist	-.091	.033	**
Variance Estimates	Variance		χ^2
Reliability (λ)	.971		
Teacher Level Variance (σ^2)	.762		
School Level Variance (τ_{00})	.302		95.61
ICC (ρ)	.284		
Final Teacher Level Variance (σ^2)	.448		
Teacher Variance Explained	41.2%		

* $p < .05$ ** $p < .01$ *** $p < .001$

Beliefs of teachers in schools with an elementary school Mathematics Specialist did not differ significantly in terms of the Traditional perspective from those teachers in the control schools, unless the teacher was highly engaged with the Mathematics Specialist. Teachers who

were highly engaged with their Mathematics Specialist had, on average, a 9.1% SD lower measure on the Traditional beliefs scale, a statistically significant difference.

Teacher Engagement in Other Professional Development—To investigate the effects of elementary Mathematics Specialists on teachers' professional development, a parallel model was fit to three dependent variables indicating whether a teacher attended a mathematics-centered grade-level meeting, whether they observed a colleague teach a mathematics lesson, and whether they attended a schoolwide mathematics instruction workshop. Since these were binary outcomes, the analyses fit hierarchical linear models with a Bernoulli distribution and a logit link function. These models return logit coefficients which, unlike probabilities, are additive. These logit coefficients were converted into the probabilities that teachers would engage in different types of professional development based on a number of characteristics. These characteristics included four individual variables and three school-level variables. The four teacher-level measures were: the final Traditional and Making Sense beliefs scales, a binary indicator for novice teachers (1-2 years of experience), and a binary indicator for African-American teachers. Originally, all races were included in the model, but only African-American teachers showed differences, and the proportion of teachers whose race/ethnicity was neither White nor African-American was very small. The models also included three school-level binary variables (Treatment schools, Title I schools, and Title I Treatment schools).

Table 10 presents results of these analyses for three dependent variables (Attending mathematics-centered grade-level meetings; Observing a colleague teach mathematics; Attending a school-based mathematics workshop) with three sets of columns. The first and second columns of statistics in each set present the logit coefficients and the standard errors associated with each variable; significant coefficients are marked in the table with asterisks. The third column in each set presents the probabilities for each variable, calculated by summing the logit coefficients that apply for each significant coefficient. For example, in the middle set of statistics, the probability of observing a colleague for a teacher in a treatment school is calculated by summing the logit coefficients for the intercept and the Treatment variable, and converting that sum to a probability. Likewise for teachers in a Title I Treatment school, which is an interaction term, the coefficients for all the constituent variables—Treatment, Title I school, and Title I Treatment school—are added to the intercept and then converted into a probability. Thus, while positive logit coefficients indicate increased probabilities and negative logit indicated decreased probabilities, only summed probabilities can be compared directly to the intercept. These probabilities indicate the relative power of each significant variable, and are not recorded for non-significant variables.

Table 10
Effect of Elementary Mathematics Specialists on the
Probability of Teachers Engaging in Professional Development

	Attend Mathematics-Centered Grade-Level Meetings			Observing a Colleague Teach			Attend a School-Based Mathematics Workshop		
	Coefficient	Standard Error	Summed Probability	Coefficient	Standard Error	Summed Probability	Coefficient	Standard Error	Summed Probability
Intercept	1.504 ***	.116	.818	-.107	.079	.473	.396 ***	.078	.598
Treatment School	1.090 **	.384	.93	.758 ***	.270	.657	.747 **	.262	.758
Title I School	.320	.388		.843 **	.279	.676	.896 **	.273	.785
Title I Treatment School	-1.028	.544		-.761	.383		-1.100 **	.376	.719
Making Sense Beliefs	.326 ***	.049	.860	.270 ***	.041	.541	.256 ***	.041	.657
Traditional Beliefs	.021	.057		.154 ***	.044	.512	.132 **	.045	.629
Novice Teacher (1-2 yrs)	-.398 ***	.121	.752	.345 ***	.103	.559	-.134 ***	.103	.561
African-American Teacher	.283 *	.121	.857	.405 ***	.100	.574	.308 **	.103	.669

* $p < .05$ ** $p < .01$ *** $p < .001$

Each of the three intercept values in Table 10 indicates the average probability that a teacher engaged in a given category of professional development. Teachers were highly likely to attend a mathematics-focused, grade-level meeting (81.8% probability), were more likely than not to report attending a school-based mathematics workshop (59.8%), and slightly less than likely to observe a peer teach a lesson (47.3%). Individual teacher beliefs influenced teachers' engagement in professional development, as teachers with a Making Sense perspective were somewhat more likely to engage in all types of professional development, while Traditional beliefs evidenced a slightly lesser influence, with no significant impact on grade-level meeting attendance. During their first two years of teaching, teachers were significantly more likely to observe other teachers teaching (55.9% - 47.3% = 8.6%), but slightly less likely to take advantage of other professional development opportunities. African-American teachers were consistently more likely to attend all types of professional development sessions.

School-level variables consistently influence attendance at professional development sessions. Teachers in schools with elementary Mathematics Specialists were more likely to attend all three forms of professional development as compared to teachers in control schools. The likelihood of peer observations and local-school workshop attendance was higher in Title I schools by 20.3% and 18.7%, respectively. There were no differences in the likelihood of teacher attendance at mathematics grade-level meetings or peer observations between Title I and non-Title I Treatment schools. However, teachers in Title I schools with an elementary Mathematics Specialist were slightly less likely to attend school mathematics workshops, though still 12.1% (71.9% - 59.8%) more likely to attend than teachers in control, non-Title I schools.

Discussion

This study found that elementary Mathematics Specialists had a significant positive impact on student achievement over time, but this effect only emerged as knowledgeable Specialists gained experience and as schools' instructional and administrative staffs learned and worked together. Simply allocating funds and then filling the position of an elementary Mathematics Specialist in a school will not yield increased student achievement. The Specialists in this study influenced the beliefs about mathematics teaching and learning held by the teachers with whom they were highly engaged, increasing a Making Sense perspective and diminishing a Traditional perspective. Further, teachers in the schools with an elementary Mathematics

Specialist were more likely to engage in other forms of available professional development addressing mathematics content and pedagogy than were teachers in the control schools.

As inferred from the PDA data, the Specialists in this study were more likely to focus their coaching efforts on individual teachers, rather than on leading grade-level planning teams. The time that Specialists had to coach individual teachers seemed to diminish over the three years of PDA data collection, while the time that they devoted to supporting student assessment demands increased. While this pattern was evident across schools in each of the five cooperating school districts, each year there were one or two schools in urban districts (identity of schools varied by year) where not only was time allocated to managerial aspects of assessment, but increased time was also spent working with students without an observing teacher. It is recognized that if a teacher is absent on a given day, many administrators will request that the school's Mathematics Specialist, rather than the assigned substitute teacher, teach the mathematics lesson to the absent teacher's class on that day. However, this practice is not unique to urban districts. Thus, that is not likely to be the explanation for this pattern. It may be that if a Specialist devoted an unusual increase in time for assessment and independent instructional roles in a given year, it was in response to her local school's administrative expectations.

All of the elementary Mathematics Specialists in this study were responsible for coaching teachers of mathematics in their schools, but they also had programmatic responsibilities. These responsibilities included assisting the administrative staff in interpreting assessment data, ensuring that their schools' curriculum was aligned with district and state standards, working to foster home/school/community partnerships focused on students' learning of mathematics, and collaborating with their principal to support a schoolwide mathematics program.

The Specialists who were the subjects of this study engaged in substantive academic coursework that was designed to foster and support their transition to the position of whole-school elementary Mathematics Specialist. As such, the results herein should not be generalized to other settings where an experienced teacher is simply named as the school-based Mathematics Specialist or coach with little or no prior professional development addressing the responsibilities and expertise presumed of elementary Mathematics Specialists or coaches. Further, there have been recent recommendations that schools employ a specialized teacher model, particularly in the upper elementary grades, in which all students receive their mathematics instruction from a mathematically well-prepared teacher [13]. While this study and many of the school districts

cooperating in this study used the term “mathematics specialist,” the model of a specialized teacher for instruction was not implemented in this study.

Acknowledgment

This article was developed with the support of the National Science Foundation, Grant #ESI-0353360. The statements and findings herein reflect the opinions of the authors and not necessarily those of the Foundation.

References

- [1] F. Fennell, “President’s Message: We Need Elementary School Mathematics Specialists NOW,” *NCTM News Bulletin*, **43**(4) (2006) 3.
- [2] *Adding It Up: Helping Children Learn Mathematics*, National Research Council, Washington, DC, 2001.
- [3] B. Neufeld and D. Roper, “Coaching: A Strategy for Developing Instructional Capacity,” *Education Matters*, Cambridge, MA, 2003; Internet: <http://www.annenberginstitute.org/Products/Coaching.php>
- [4] P.F. Campbell and D.Y. White, “Project IMPACT: Influencing and Supporting Teacher Change in Predominantly Minority Schools,” in E. Fennema and B.S. Nelson (eds.), *Mathematics Teachers in Transition*, Lawrence Erlbaum and Associates, Mahwah, NJ, 1997.
- [5] R. Marzano, T. Walters, and B. McNulty, *School Leadership That Works: From Research to Results*, Association for Supervision and Curriculum Development, Alexandria, VA, 2005.
- [6] J. York-Barr and K. Duke, “What Do We Know about Teacher Leadership? Findings from Two Decades of Scholarship,” *Review of Educational Research*, **74**(3) (2004) 255-317.
- [7] B. Joyce and B. Showers, “Improving In-Service Training: The Messages from Research,” *Educational Leadership*, **51**(2) (1980) 57-61.
- [8] S. Loucks-Horsley, C.K. Harding, M.A. Arbuckle, L.B. Murray, C. Dubea, and M.K. Williams, *Continuing to Learn: A Guidebook for Teacher Development*, The Regional Laboratory for Educational Improvement of the Northeast and Islands and The National Staff Development Council, Andover, MA, 1987.
- [9] M. McGatha, *Coaching Bibliography*; Internet: <http://louisville.edu/education/research/centers/crmstd/mcr/math-coaching-biblio.pdf>.
- [10] D. Foster and P. Noyce, “The Mathematics Assessment Collaborative: Performance Testing to Improve Instruction,” *Phi Delta Kappan*, **85**(5) (2004) 367-374.
- [11] J.A. Ross, D. McDougall, C. Bruce, S.B. Jafaar, and J. Lee, “A Multi-Dimensional Approach to Mathematics In-Service,” in *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, University of Toronto, Ontario Institute for Studies in Education, Toronto, Ontario, 2004.

- [12] J.A. Ross, D. McDougall, A. Hogaboam-Gray, and A. LeSage, “A Survey Measuring Elementary Teachers’ Implementation of Standards-Based Mathematics Teaching,” *Journal for Research in Mathematics Education*, **34**(4) (2003) 344-363.
- [13] *Foundations for Success: Findings and Recommendations from the National Mathematics Advisory Panel*, US Department of Education, National Mathematics Advisory Panel, Washington, DC, 2008 (ED 00424P).

WHAT WE ARE LEARNING ABOUT THE ELEMENTARY MATHEMATICS SPECIALIST'S ROLE: SOME REFLECTIONS ABOUT MATH COACHING

J. WHITENACK and A. ELLINGTON

*Dept. of Mathematics and Applied Mathematics, Virginia Commonwealth University
Richmond, VA 23284-2014*

A Day with a Mathematics Specialist

The elementary Mathematics Specialist met us at the school office and whisked us off to her office somewhere down one of the long hallways. After weaving through the corridors, we entered her office, which she shares with other specialists (e.g., special educator, child psychologist, etc). As we proceeded to her work area, she told us about recent events. She has a new principal, one that supports her work much more than the previous one did. She welcomes the support and direction that the new principal plans to take. As she continues to recount the new initiative that her principal has established around the Mathematics Specialist's role this new school year, she suddenly realizes that it is time to begin her morning rounds to different classrooms. Every forty-five minutes or so, we will visit several classrooms: a first grade classroom, followed by a third grade classroom, followed by a special education classroom, etc. As we enter the first grade classroom, the Mathematics Specialist hits the floor running. She follows the classroom teacher's lead regarding the pacing of the lesson. She interjects questions to further clarify a student's thinking during the whole class discussion. She communicates with the classroom teacher briefly as the students begin their small group work. She then speaks with different students, listening and responding as they explain their thinking. She stops at one student's desk because he is not working on his assignment, nor does the regular teacher expect him to since he lags at least two years behind his fellow classmates in all subjects. The Mathematics Specialist adjusts the activity, and works for several minutes with this student before moving on to another student's desk. After about thirty minutes, we proceed to the next classroom. There, we observe a similar routine. After three classroom visits, we bid good-bye because we must visit a different school building. As we leave this building, we wonder how the Mathematics Specialist works at this pace for hours on end with such energy, purpose, and confidence. It does not matter which school building we enter, what leadership activities we observe, our question is always the same: What motivates, energizes, and sustains the Mathematics Specialist as she engages in her daily work? The answer comes immediately: the students.

Observation and Interviewing Process

The above account provides a glimpse of a Mathematics Specialist's daily schedule. Some of her many responsibilities throughout the school day include teaching or co-teaching lessons at different grade levels, assessing and "remediating" students' mathematical difficulties, providing materials for teachers, preparing teachers and their students for the upcoming quarterly benchmark tests, as well as the end-of-the year state-mandated tests, or planning and participating in math coaching sessions with teachers in the school building [1].

Over the past few years, we have worked closely with six Specialists to develop a better understanding of what their Specialist roles entail. To accomplish this task, we visited the school buildings in which they worked and observed their daily routines as Specialists. We also conducted interviews with them to get first-hand information about their daily work. Sometimes, we interviewed them before observing their work with students or teachers. At other times, we interviewed them after we observed their school-based activities.

We have begun to synthesize this information to provide a more detailed account of the scope of their work. Because coaching is one of the more important aspects of their roles, we have focused our efforts on understanding their coaching roles. Following Fullan and others, we believe that teacher learning must occur in the teachers' own classroom as they work with students [1-4]. Thus, the Specialist's role as a coach is particularly important because she has the opportunity to support the classroom teacher's learning. As stated in the National Council of Supervisors of Mathematics' *The PRIME Leadership Framework*, "A single mathematics education leader can have an incredible impact on the development and effectiveness of others" [5]. So she might affect teaching practice on a small or large scale as she works side by side with teachers to enact research-based practices, such as those ascribed to by the National Council of Teachers of Mathematics [6].

The Specialist's Evolving Role

In our discussion, we outline several themes that have emerged in our work with these six (female) Specialists. These themes are based on examples taken from their daily work in their respective school buildings. The themes are composites of sorts and do not represent one particular Specialist's experience. So as we present each theme, we might use examples taken from several of the Specialists' work on any given school day.

The themes that we highlight relate to how the Specialist defines her role, with whom and where she works, her continued learning on the job, and the importance of coaching teachers. Although these themes are related to one another, we highlight those issues that are unique to a particular theme. After we do so, we provide an example to illustrate how issues related to one of the themes, coaching teachers, is enacted as a Mathematics Specialist works with a new teacher.

Theme 1: The Specialist Defines Her Own Role within the School Building—As the Specialist has conversations with teachers, the building principal and others who may help to shape her role in the school building, she makes choices about what aspects of her role are and are not negotiable. How can she effectively support teacher and student learning? For instance, she might be asked to formally evaluate a classroom teacher’s mathematics instruction. Should she? If the principal asks her to join her as she conferences with the teacher, how might the Specialist respond? She realizes that if she serves in a formal role as an evaluator, she cannot effectively work collaboratively with the classroom teacher. Similarly, she is aware that if she only works with students (e.g., pulls students out of the classroom), she will not be able to work closely with the teacher to positively affect the teacher’s instructional practice. Further, she must find a common ground for how she addresses, manages, and supports teachers as they prepare for state-mandated assessments. Thus, the role that she establishes is hinged on the extent to which she can effectively support teacher and student learning.

Theme 2: The Specialist Is a Life-Long Learner—The Specialist continues to learn on the job. In fact, because of the nature of her work, she will need to learn parts of her craft as she works with teachers, students, and school district personnel. She will continue to develop a rich and deep understanding of the mathematics that is covered in the elementary school curriculum. She also will continue to make connections to develop an intricately woven web of ideas about teaching, about student thinking, and about how to best support teacher learning. As such, she will draw on old as well as new resources to plan and implement professional development activities for her teachers, principal, district personnel, and possibly others in the community in which she works.

Theme 3: The Specialist’s Work Is Situated in a Living and Breathing System—As a result, change will occur. In fact, the Specialist will adapt to a wide range of changes that may affect her work from one year to the next. She will move her office, perhaps every year. So, she will need to pack and unpack, reorganizing all the mathematics education materials each time. She may have a different principal after only working in the school building for a few years. So, she will need to re-establish her role as a Mathematics Specialist as she works with this new principal. In

addition, in any given year, she may have an unusually large turnover of teachers. Perhaps there will be twenty new teachers in her school building. Most of these teachers will have never taught mathematics using research-based practices. Therefore, she will begin to work with them immediately as the school year unfolds. Further, because her school did not meet Adequate Yearly Progress (AYP) the previous year, the principal may require her to work with students in a pullout program to ready them for the quarterly and the state-mandated end-of-the-year tests. In any given year, she will face these and other changes that provide unforeseen challenges. In some cases, the results of the work that she did the previous year will no longer be useful because of these changes, so she will need to adapt her goals, her practices, etc. as she faces these changes from year to year.

Theme 4: Part of the Specialist’s Work Is to Develop Mathematics Teacher Leaders—One of the biggest challenges that the Specialist faces is identifying and building relationships with other teachers who will eventually become teacher leaders in the school building. So, she makes deliberate choices regarding with whom she works. By choosing to work with teachers who have the potential to become leaders in the school building, she begins to craft a plan for affecting mathematics instruction on a broader scale. Her aim is to essentially “work herself out of a job.” To accomplish this, she will support colleagues as they move into these leadership roles. For instance, she might encourage a colleague to conduct a workshop or to observe other teachers’ mathematics instruction or even co-teach a lesson with another teacher. The Specialist might work collaboratively with this potential leader, or she might teach in this colleague’s class while her colleague works with other teachers.

As she works to build capacity, there will be others who will be unsupportive of her work. Hence, she will likely meet a number of different obstacles that might be characterized as meeting resistance. Some will be direct with their lack of support for the Specialist’s work. Others may simply choose not to participate and will encourage their colleagues to do the same. As a result, it will take time to develop these productive, collaborative relationships with teachers in her school building.

Theme 5: The Specialist Actively Engages in Content-Specific Coaching—The Specialist engages in content-specific coaching in order to affect teachers’ practice which in turn may support student learning. When working with teachers, the Specialist will have clearly defined goals on which to focus a coaching session. She and the teacher will talk about the important mathematical ideas that they will highlight during the lesson. In some instances, as they plan,

they may not anticipate all of the issues that will surface during the lesson. As such, the Specialist will have the opportunity to learn along with the classroom teacher about students' methods, misconceptions, etc. What is important is that the Specialist finds ways to address key mathematical ideas around students' mathematical thinking and misconceptions. She will likely address these ideas during the planning session. If not, she will have the opportunity to do so after the lesson, either formally or informally. *When* she addresses these issues pales in comparison to *how* she chooses to capitalize on these opportunities for teacher learning.

Discussion

As we stated earlier, these five themes are not mutually exclusive. To illustrate this point, let us consider *how the Specialist establishes her role* might be related to *the extent to which she engages in content-specific coaching*. Suppose the Specialist works collaboratively with teachers during the first part of the school year. In particular, she and one of the grade-level teams plan a series of lessons that cover one of the strands in the curriculum. Each of the teachers will teach a mini-lesson to one of the groups of students as the students rotate from classroom to classroom throughout the morning. During this "math event," the Specialist moves from classroom to classroom to support teachers as they implement these mini-lessons. If necessary, she might co-teach part of the lesson to support one of the new teachers who is implementing this type of lesson for the first time. This event is one of many professional activities that the Specialist engages in as she works with teachers during the first part of the school year. By way of contrast, during the second part of the school year, the Specialist will not be able to engage in these types of activities. At the principal's request, she must prepare third, fourth and fifth graders for the state-mandated standardized tests that the students will take in May. So during the second part of the school year, the Specialist has a different set of responsibilities than she had during the first part of the school year. She must develop practice tests and then grade those tests to identify students who did not score 100% on any portion of the test. Once she has identified these students, she will develop lessons for them that target only those concepts that they missed on the practice test. After implementing this intervention, she will then administer a second practice test to determine if these students mastered the concepts and skills. Because she needed to redirect her focus to that of preparing for the end-of-the-year, state-mandated assessments, she is unable to work with individual teachers, and more generally, to engage in coaching activities. As this example illustrates, her role as a coach is directly affected by what the principal perceives to be the Specialist's role during the second part of the school year.

In the next section, we highlight the Specialist's role as a coach to illustrate aspects of the Mathematics Specialist role as it might be enacted. Because of the important role that coaching can play in affecting teacher and ultimately student learning, we frame our discussion around an example taken from a coaching session between one of the Specialists, Ms. Snead, as she worked with a first-year teacher, Mr. Stark.

Coaching a New Teacher—Background

Ms. Snead, one of the six Specialists that we have followed, became a Mathematics Specialist during the second year that she was enrolled in a graduate endorsement program [7]. She has continued to work in the same building since she became a Specialist. Over time, Ms. Snead has become more selective about which teachers with whom she collaborates. During this school year, her fourth as a Specialist, she worked with Mr. Stark, a first-year teacher that recently joined the faculty after graduating from a state university. Because Mr. Stark taught at one of the grade levels that did not perform well the previous year on the state-mandated assessments, Ms. Snead has been encouraged by her principal to work with him and other teachers that taught at this grade level.

The example that we share is taken from one of our school building visits when we observed Ms. Snead plan and co-teach a lesson with Mr. Stark. When we visited with Ms. Snead and Mr. Stark, it was only their second coaching session, although they talked about mathematics instruction from time to time. Remarkably, even though they had only collaborated for a short period of time, they seemed to work quite well together.

The Coaching Session

A coaching session is, at least in theory, a three-part process [1]. The Specialist plans a lesson in collaboration with the teacher. During this planning session, they might explore the mathematics that is to be taught, work out logistics, and develop shared goals for their work together. As part of the process, they decide, for instance, whether the Specialist will co-teach or simply observe the lesson. After planning the lesson together, the Specialist visits the teacher's classroom to observe (and possibly co-teach) the lesson. The Specialist and the teacher meet immediately after the lesson has been taught to debrief about what happened during the lesson. For her part, the Specialist makes careful and deliberate decisions about what issues to address during this debriefing session (e.g., student learning, the teacher's practice).

Ms. Snead and Mr. Stark's Planning Session

During the planning session, Ms. Snead first helped Mr. Stark to outline the big ideas for the lesson using a lesson plan form. As they filled out the form together, they discussed the state and district standards related to the goals of the lesson. Ms. Snead recorded information on the form as they talked about these ideas. They also discussed issues related to working with English Language Learners (ELL). For example, they identified several words in the problem (e.g., “catalog,” “presents,” “Christmas,” and “spent most”) that might be problematic for students with language challenges. As the discussion continued, Ms. Snead and Mr. Stark addressed his students' estimation number sense. During this conversation, they talked about how students might use a front-end strategy (i.e., round numbers to the nearest 1,000, 100, and 10) to solve problems. Ms. Snead even illustrated how students might use this strategy to estimate 10,542. By the end of the twenty-minute session, Ms. Snead and Mr. Stark agreed that, as students worked in small groups to solve the problem, they would monitor students' progress. They also agreed that when they reconvened the small groups for a whole-class discussion, Mr. Stark would highlight some of the different methods that the students used to solve the problem. Although they did not explicitly talk about what Ms. Snead's role might be during the lesson, it was understood that Mr. Stark would teach the lesson.

On a side note, Ms. Snead and Mr. Stark planned the lesson, but they did not plan for what they might do if students only derived exact answers as they solved the problem. As we summarize the lesson, note that this issue surfaced as an important one.

The Lesson

Mr. Stark began the lesson by posing problems that involved using different denominations to make \$0.78, \$1.09, \$0.98, and \$1.23. During this part of the lesson, several students talked about which coins they used to make the specified amounts. For each problem, a student explained that he used the largest coin (the quarter) until he could not use this coin anymore and continued to use the next lower denomination until he had made the total amount. As a student explained his ideas, Mr. Stark recorded the answers by writing each type of coin that the student used. For example, for \$0.98, a student explained that he used 3 quarters, 2 dimes, and 3 pennies. As the student gave his answer, Mr. Stark recorded QQQDDPPP to represent each of the coins.

After this brief discussion, Mr. Stark introduced the following problem using the overhead projector (see Figure 1).

You have saved your money all year to buy Christmas presents for your family and closest friends.
 You have a total of \$26,462. You need to buy 4 presents.

Below is a list of gifts from your favorite catalog (they do not charge tax or shipping). What gifts could you buy? You want to spend most of your money and you have to hurry.

(In case they are out of one or more items, please prepare at least two possible lists.)

Life-Size Teddy Bear	\$1,256
Miracle Kitchen Cleaner	\$14,589
Custom Computer Game	\$10,542
Picture-Tube Telephone	\$7,499
Fancy Paints and Brushes	\$3,611
Ski Trip for Two People	\$19,653
Dinner for Two in Boston	\$15,576
Big-Screen TV with VCR	\$2,734
Dessert of the Month Club	\$4,510
Movie of the Month Club	\$6,500

Figure 1. Mr. Stark posed the “Estimation Problem.”

As he posed the problem, he paused before reading \$26,462 and asked students to read this number. After he asked this question, Ms. Snead asked the students to talk with their partners about how they would read this number. Mr. Stark continued to pose the rest of the problem once the students had shared their ideas. While Mr. Stark began to read the part of the problem about “favorite catalog,” Ms. Snead moved to the front of the room and drew a logo of a popular toy store. She also asked students to talk to a fellow classmate about the meaning of the word “catalog.” When Ms. Snead and the students talked about the meaning of the word “catalog,” Mr. Stark went to his desk and picked up a large catalog. As the discussion continued, he referred to this catalog while he and the students finished reading the problem.

Mr. Stark then asked the students to begin working on the problem. At this point, Ms. Snead spoke with Mr. Stark. Afterward, he immediately mentioned to the students that their answers did not need to be exact (since they were estimating the total spent). The students continued to work on the problem for the next twenty minutes. During this time, Mr. Stark and Ms. Snead talked with different groups of students. They also talked to each other briefly several times as they moved from group to group.

Ms. Snead and Mr. Stark conferred again right before they reconvened the students for the whole-class discussion. Mr. Stark and Ms. Snead facilitated a lively discussion about one group's strategy. Students in this group explained that they had added $\$15,000 + \$1,000 + \$7,000 + \$3,000$ to derive an estimate of $\$26,000$. As they explained their strategy to their classmates, Mr. Stark recorded their method on the whiteboard.

In the next section, we highlight parts of interviews that we had with Mr. Stark and Ms. Snead, respectively, immediately following the lesson. We were particularly interested in finding out what they had talked about during the lesson. Did Ms. Snead provide certain kinds of support for Mr. Stark during the lesson? What types of support did she offer? Did Mr. Stark ask Ms. Snead questions about how he might proceed? By considering their comments against the backdrop of our observations during the lesson, we hoped to develop a better understanding of how these conversations might have benefited Mr. Stark and his students.

The Follow-up Interview with Mr. Stark and Ms. Snead

When we spoke to Mr. Stark, we asked him about the conversations he had with Ms. Snead during the lesson. In the excerpt that follows, he indicated what they talked about during one of their exchanges, and explained why he and Ms. Snead spoke just before he reconvened the students for the whole-class discussion about the estimation problem.

Interviewer: During the lesson at one point you and Ms. Snead were talking while the students continued to work on the problem. What did you talk about?

Mr. Stark: I asked her to go check on the group because they might be a group that I might want to share their method. We [were] trying to find a group that we could use as a presenting group...I just said go take a look at that group because it looks like they had done some estimation and she said, "Let's get [those students] up there."...

When Mr. Stark and Ms. Snead communicated at this point during the lesson, they made a decision about which students' work to highlight during the discussion. As Mr. Stark's comment suggests, he asked Ms. Snead to check one of the group's solution methods to see if she, too, agreed that they should highlight this group's method during the whole-class discussion. He went on to say that after she talked with this group, she communicated to him that she agreed with his suggestion. His next statement, "Let's get [those students] up there," is evidence of this fact. Additionally, notice that he stated, "We [were] trying to find a group that we could use as a

presenting group.” His choice of the pronoun “we” is particularly interesting. Mr. Stark stated what appeared to be one of their shared goals for the lesson.

When we spoke with Ms. Snead after the lesson, she also provided insight into the types of conversations that she and Mr. Stark had during the lesson. In fact, she mentioned another conversation that occurred earlier in the lesson. She said that after Mr. Stark sent the students off to solve the problem, she asked him to “make sure you emphasize that [the students] do not need to get the exact answer.” She also said that when she made this suggestion to Mr. Stark, he stated, “Oh yeah, I forgot about that...”

We, too, recalled this instance during the lesson, and wondered what they might have talked about. As it happens, immediately after she reminded him of this point, Mr. Stark gave some additional instructions to the students while they worked on the problem. He mentioned that they should not find an exact answer (i.e., they were to estimate). Therefore, by speaking to Mr. Stark immediately after he had finished posing the task, she made a timely suggestion that he then incorporated into the lesson in the form of additional directions to the students.

Interestingly, as in our previous example, the exchange between Ms. Snead and Mr. Stark seemed to advance the goals of the lesson. In both cases, Mr. Stark had the opportunity to possibly develop or refine certain teacher moves during the lesson. In the former, Ms. Snead supported his decision to select a particular group’s work to highlight during the whole-class discussion. In the latter, she reminded him about the intent of the activity. So in both instances, the exchanges they had advanced the goals of the lesson, albeit, in different ways.

Did Mr. Stark benefit from conversations with Ms. Snead during the lesson? We suspect that these types of exchanges were possible learning opportunities for Mr. Stark (and possibly for Ms. Snead). Mr. Stark could communicate with an in-house expert about subsequent instructional moves that he might make during the lesson.

Students also may have benefited from the collaboration between Ms. Snead and Mr. Stark. In fact, we suspect that the exchanges they had also supported his work with his students. During the lesson, for instance, Mr. Stark highlighted a particular group’s solution method. As a result, other students, most of whom had derived exact answers, had the opportunity to understand their classmates’ strategies for estimating the amount of money they could spend. So

as Ms. Snead collaborated with Mr. Stark, she may have supported not only Mr. Stark's, but also his students' learning.

These types of situations may also have been learning opportunities for Ms. Snead. She may have developed a better understanding of the extent to which she and Mr. Stark needed to discuss the purpose of the lesson as they planned together. Perhaps she presumed that they did not need to discuss the intent of the activity. Or, she may not have realized that they needed to discuss this issue during the planning session. Although we do not know which of these or possibly other scenarios might best describe Ms. Snead's situation, her decision to address the issue during the lesson seemed an important one.

Interestingly, during the interview, Ms. Snead did mention what she does when unanticipated issues surface during the lesson. She made the following statement:

It doesn't matter what the lesson is, there are things that come up that we should have talked about during the planning session. Or, "Oh I wouldn't have known to talk about this during the planning session, but I want to talk about it now."

As her comment states, when issues come up that need to be discussed with the teacher, she can address them when she converses with the teacher after the lesson. Or, perhaps Ms. Snead may mention these issues during the lesson, for instance, as she did with Mr. Stark. Additionally, because she and Mr. Stark talked about this and other issues during the lesson, she could revisit them after the lesson. Consequently, when situations occur that the Mathematics Specialist may not have anticipated, she may have opportunities to reflect on her own coaching practice. What ideas does she need to address with the teacher about students' misconceptions? Does the teacher understand what the intent of the lesson is? As the Specialist considers these and other questions, she has the opportunity to refine her coaching practice. In our example, for instance, Ms. Snead might consider why she needed to mention to Mr. Stark that the students should not compute exact answers to solve the problem. When she and Mr. Stark plan another estimation lesson, she might refer to this earlier lesson to highlight students' misconceptions. In doing so, she would continue to refine her own skills as a math coach, and at the same time, provide opportunities for Mr. Stark to refine his teaching skills.

During the interview with Mr. Stark, we also asked him what he learned from working with Ms. Snead during this coaching session. When we asked Mr. Snead this question, we

thought that he might address issues around the students' misconceptions about estimation. Instead, he responded that the most important idea that he learned was related to aligning the lesson with the pacing guide and the state standards:

Biggest thing was looking at the standards that corresponded with the lesson and think about [how]I had addressed [them] previously. I didn't think about how I had addressed it before today and now that I am talking to you, I realize that I haven't addressed it in weeks really...I see that it is something that I really need to go back to. So that is something that I haven't been thinking about in terms of the standards in each individual lesson. I haven't been thinking about how 3.4 corresponds to the lesson I am doing today...And the standard that corresponds to the pacing guide...think about that more, I can keep track of things that I [am] hitting and things that I am not.

As Mr. Stark's comment suggests, he thought that his discussion about the pacing guide during the planning session was helpful. He could (and should) use these documents to guide his planning among other things. As he stated, he could "keep track of things that I [am] hitting and things that I am not."

Notice also that Mr. Stark mentioned that the "biggest thing" he learned was related to using the pacing guide to frame lessons. What else did he learn working with Ms. Snead? As we have suggested previously, Mr. Stark had additional opportunities to learn or refine ideas about teaching estimation during his conversations with Ms. Snead during the lesson. In fact, during the lesson, Ms. Snead told one of the researchers that he mentioned that he should have posed a different task at the beginning of the lesson—an estimation task—before introducing the problem solving activity. While he did not mention this and possibly other issues during our interview session, he may have had other opportunities to reflect on and learn about his practice before, during, and after this lesson.

Follow-up Interview Discussion

We use this example to illustrate how interrelated the learning opportunities are for the teacher, the Specialist, and the students. In our example, the issues that did not surface during the planning session were ones that became important as Ms. Snead and Mr. Stark made decisions during the lesson. Whereas Ms. Snead but not Mr. Stark addressed students' misconceptions about estimating during the interview session, we suspect that this was one of the key ideas they might explore during subsequent planning sessions.

Our example is also instructive to Mathematics Specialists as they work with first-year teachers. One of the ways the Specialist can help the teacher is *to move beyond* discussions about logistics, pacing guides, and state standards. Although it is important to have these types of discussions with new teachers, it is also important for the Specialist to address ideas around students' misconceptions, representing students' thinking, etc. Because it is impossible to address every important issue during planning sessions, the Specialist must still choose the focus of the planning session. If important issues that were not addressed during the planning session do surface during the lesson, she has additional opportunities during and after the lesson to highlight these ideas [1].

Final Comments

In our discussion, we have highlighted five themes that have emerged as a consequence of our work with Specialists. For emphasis, we have addressed issues that are unique to each of the five themes. We have also illustrated how these themes might be related to one another. In this final section, we address several broader issues about coaching.

We have also illustrated how the Specialist might affect not only the classroom teacher's learning, but also his students' mathematical learning. As these examples illustrate, the Specialist may make different types of contributions during the lesson. Some of the Specialist's contributions may directly influence or reorient the teacher during the lesson. Others may simply support the classroom teacher as he makes instructional decisions. In either case, the Specialist must be quite flexible as she moves in and out of these different types of situations.

Additionally, we have shown that coaching may provide opportunities not just for teachers, but also their students to build new understandings. In Mr. Stark's classroom, for instance, Ms. Snead played an active role in supporting him as he made decisions so that his students could explore particular mathematical ideas. Hence, the teacher and the students, albeit in different ways, may benefit from the Specialist's role as a math coach.

The Mathematics Specialist faces many challenges as she establishes her role in the school building, particularly as she enacts her role as math coach. She needs to "convince" the school building principal and possibly administrative personnel that coaching is a critical part of her job. She must work hard to find blocks of time so that she can work effectively as a math coach (e.g., plan, co-teach/observe *and* debrief about the lesson). In most cases, the six Specialists that we observed rarely had opportunities to engage in all three phases of a coaching

session. This issue is problematic from our point of view, and yet it is presently one of the realities of school life. Ideally, it would be more beneficial for all (Specialists, teachers, and their students) if Specialists could enact all three phases of the coaching process in a single block of time. Ms. Snead for instance, will be hard pressed to have more than a brief, follow-up conversation with Mr. Stark about this estimation lesson. She will likely speak with him as he takes his students to the library or as they walk students to the buses at the end of the day. As such, Mr. Stark may have fewer opportunities to benefit from these coaching sessions.

Because teachers' classrooms are the sites for their learning, we must continue to press for professional development that best serves teachers and their students. Mathematics Specialists with whom we have worked seem to know much more about mathematics instruction in their school building, the teachers' instructional practice, the mathematics curriculum and research-based practices than most individuals in their school building, and possibly in their school district. Our observations align with Amy Morse's views about math coaches' roles in school districts (personal communication, July 2009). Her seminal book, *Cultivating a Math Coaching Practice: A Guide for K–8 Math Educators*, in turn, sheds light on the crucial ways that math coaches can affect teachers' mathematics instruction [4]. They have the opportunity to effectively support reform recommendations that have been made by the mathematics and mathematics education communities. It behooves us to understand and to support their important work.

References

- [1] L. West and F.C. Staub, *Content-Focused Coaching: Transforming Mathematics Lessons*, Heinemann, Portsmouth, NH, 2003.
- [2] M. Fullan, "Leading Professional Learning," *The School Administrator*, **10**(63) (2006) 10-14.
- [3] J. Knight, *Instructional Coaching: A Partnership Approach to Improving Instruction*, Corwin, Thousand Oaks, CA, 2007.
- [4] A. Morse, *Cultivating a Math Coaching Practice: A Guide for K-8 Math Educators*, Corwin, Thousand Oaks, CA, 2009.
- [5] *The PRIME Leadership Framework: Principles and Indicators for Mathematics Education Leaders*, National Council of Supervisors of Mathematics, Denver, CO, 2008.
- [6] *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 2000.
- [7] Virginia Mathematics and Science Coalition website, Internet: <http://vamsc.org>.

A MATHEMATICIAN’S OVERVIEW OF THE VIRGINIA ELEMENTARY MATHEMATICS SPECIALIST PROGRAM

L.D. PITT

*Dept. of Mathematics, University of Virginia
Charlottesville, VA 22904*

Abstract

This article discusses the mathematics component of the Mathematics Specialist master’s degree program in the “Virginia Mathematics Specialist Project” (VMSP). It includes my personal views on the significant mathematical knowledge and skills that Mathematics Specialists need, the mathematics that is taught in the Mathematics Specialist courses, and my thoughts on what appear to be the substantial mathematical abilities and aptitudes that are required by successful Mathematics Specialists in their work. The interpretations I present are highly personal and are undoubtedly dependent on my personal history, a short description of which is given (see Appendix A).

Background

I use “Virginia Mathematics Specialist Project” (VMSP) as a term covering the work done in Virginia over a seven-year period with a sequence of three Virginia MSP Specialist grants and two, five-year NSF projects: the TPC project, “Mathematics Specialists in K-5 Schools: Research and Policy Pilot Study”; and, the NSF Institute project, “Preparing Virginia’s Mathematics Specialists.” All this work was done under the umbrella of the Virginia Mathematics and Science Coalition (VMSC). The partnerships included six Virginia Institutes of Higher Education (IHE), the University of Maryland, and forty-five Virginia school divisions.

Mathematical Proficiency for All

The VMSP has led the effort to implement Mathematics Specialists in Virginia. The project has had three notable successes. The Commonwealth of Virginia has established a K-8 Mathematics Specialist endorsement. Eight universities have established Mathematics Specialist master’s degree programs with 21-credit hours of common courses. Five years of research has now been completed and is discussed elsewhere in this issue. This article is focused on the mathematical core of the VMSP’s master’s degree programs, a sequence of five mathematics courses that each student takes. The courses are:

- *Numbers and Operations (N&O)*;
- *Rational Numbers and Proportional Reasoning (RN&PR)*;
- *Algebra and Functions (A&F)*;
- *Probability and Statistics (P&S)*; and,
- *Geometry and Measurement (G&M)*.

These courses align well with the content strands of elementary school mathematics as discussed in such documents as the National Council of Teachers of Mathematics (NCTM) *Principles and Standards* [1] and the Conference Board of the Mathematical Sciences (CBMS) report *The Mathematical Education of Teachers* [2]. The project started with only four courses. The *RN&PR* course was added when our experiences showed that our Mathematics Specialist students needed additional work with fractions and rational numbers that went significantly beyond that which was provided in the other four courses.

The Virginia endorsement is a K-8 Mathematics Specialist endorsement, but the program that I describe here is a K-5 program. There were many instances that arose in the program development where the breadth of the mathematics covered was limited in order to reach greater depth in the K-6 mathematics. The overall goal of the sequence of these courses is to provide future Mathematics Specialists with a *profound understanding* of the mathematics that is taught in our elementary schools. I use the term “profound understanding” to convey a significantly deeper understanding than the procedural competency often associated with mathematics courses. The term is borrowed from Ma’s *Knowing and Teaching Elementary Mathematics*, but our usage here does not align precisely with it [3]. Our goal for these courses derives from our understanding of successful learning of school mathematics. This understanding aligns nicely with that presented by the *Mathematics Learning Study Committee* in their 2001 NRC report *Adding It Up* [4]. The term the committee used to designate successful mathematics learning was “mathematical proficiency.” It consists of five interwoven strands:

- Conceptual understanding;
- Procedural fluency;
- Strategic competence;
- Adaptive reasoning; and,
- Productive disposition.

A valuable visual representation of the complex nature of the relations between these strands was presented in the form of a braid (see Figure 1).

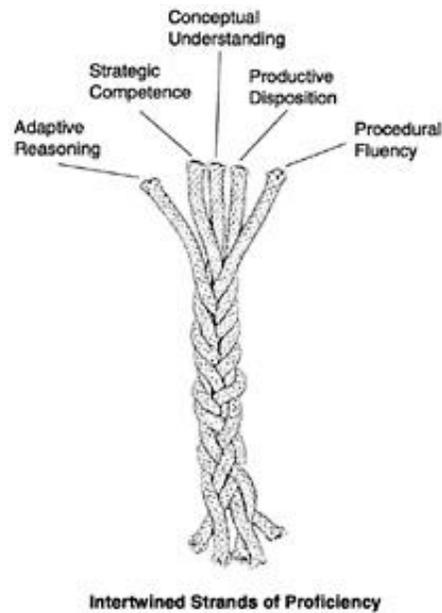


Figure 1. The five strands of mathematical proficiency represented by a braid.

A thorough discussion of mathematical proficiency is given in *Adding It Up* [4]. Of course, a precise definition of mathematical proficiency and how to assess it is problematic and there is no universal agreement on this [5]. For our purposes, it suffices to observe that this vision of school mathematics with its five braided strands is far more complex than a view which sees procedural fluency as the primary goal of school mathematics. Preparing teachers who can nurture the development of mathematical proficiency in their students and preparing Specialists who can support teachers in these efforts is also vastly more complex than preparing teachers whose goal is procedural fluency.

I wish to contrast mathematical proficiency for all with what I believe was the *standard model* for mathematics education when I was a student in the 1950s. Then, it was widely

believed that students naturally rose to the level of their mathematical talent. Some students could succeed with fractions and others could not. Fewer students could succeed with Algebra I and very few with geometry. This ascending ladder continued through graduate school and beyond. It was believed that little, if anything, could be done to counter the students' natural upper limits. As a result of these beliefs, the instructors' responsibilities were sharply limited.

This model was also reflected in a recurring event that I observed in my early years as a mathematician. Senior colleagues frequently expressed the opinion that they doubted they had ever succeeded in teaching anyone other than the few students who were so talented that they practically did not require an instructor. These remarks can still be heard in mathematics departments, but not as frequently as they once were. With this model, the instructor's primary responsibilities were to challenge the students and maintain standards. With the very best students, this model was successful, but it failed with most students. This model was never made explicit or official. My interpretation of it is based on conversations and observations of mathematics teaching that I saw practiced. Others may wish to compare my observations with their own.

The goal of mathematical proficiency for all aspires to a student population composed of confident and capable problem solvers with substantial procedural and technical proficiency. This represents a dramatic change from the apparent educational goals of sixty years ago. When only the best students were expected to succeed in mathematics, it was possible to function with a relatively small pool of highly qualified mathematics teachers. This is no longer the case. It is impossible to overemphasize the impact of this change. It drives much in contemporary mathematics education and the development of the Mathematics Specialist concept in particular.

The type of knowledge and understanding that teachers require to nurture the development of mathematical proficiency in their students is not well understood, or at least it is not well documented in the literature with which I am familiar. However, significant progress has been made in recent years toward sketching the outlines of this knowledge. In this context, I mention the work of Liping Ma and her concept of "Profound Knowledge of Fundamental Mathematics" (PKFM) [3]. Even more significant is the ongoing, large scale project on "Mathematical Knowledge for Teaching" (MKT) of Ball, Hill, Bass, and their collaborators which aspires to being able to effectively assess this knowledge [2, 6-8]. We can assert with confidence that there is a tremendous gap between the knowledge and skills possessed by typical

elementary teachers today and the knowledge and skills they would require to foster the development of mathematical proficiency in all students.

Mathematics Specialists and Their Mathematical Needs

In the Virginia project, the Mathematics Specialist's job is seen as a way to strengthen teaching practice by providing school-based mathematics support to teachers and building-level administrators. The VMSP envisions Mathematics Specialists as a primary resource for addressing the knowledge gap that exists between the current reality in our schools and what we believe is required by our students' teachers. I will now sketch my understanding of the mathematical knowledge that Specialists will need to reach this goal.

The mathematics that is covered in the VMSP program and is taught in our schools is described in the course titles. Teachers and Specialists require an understanding and familiarity of this material that includes all aspects of mathematical proficiency and large amounts of mathematical flexibility, PKFM and MKT. A glimpse into what this means is given with a few examples. Specialists must be skilled in: interpreting students' mathematical work, both written and verbal; recognizing which solutions are valid and which are not; and, having informed opinions on what a given student knows and what the next steps are. They must have a deep knowledge of how children learn mathematics and know when specific pedagogical moves are developmentally appropriate. Teachers constantly choose from a variety of representations and explanations when teaching mathematics. In the best of circumstances, these choices are based on an understanding of the students' knowledge and learning styles and the teacher's knowledge of the strengths and weaknesses of the competing explanations and models. In fact, few teachers have this skill set.

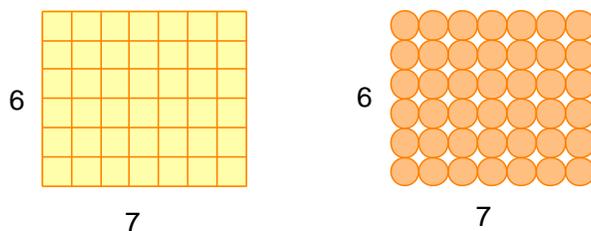
Mathematics Specialists become their schools' mathematics authorities and it is essential that they have the kind of knowledge referred to above at a much deeper level than is within reach of our teachers. A very long list of examples could be given of places where a profound knowledge of mathematics is essential for the Specialist. For example, in elementary school mathematical fallacies are quite frequently taught as fact. The errors range from the obvious to the subtle. Specialists must be able to recognize what is mathematically correct and what is false. They must be able to discuss and explain these issues with teachers, administrators, and parents. They must be able to advise and lead on issues of mathematics assessment, mathematics

curriculum, mathematics special education, mathematics for ESL students, and mathematics for the gifted and talented.

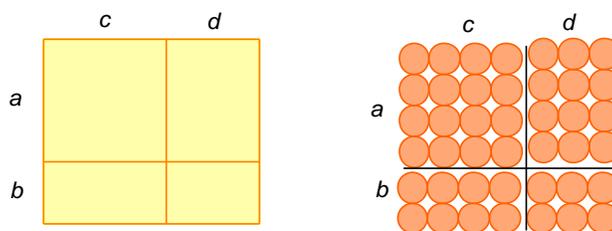
The braided strands in the illustration of mathematical proficiency are much too simple to capture the situation for Specialists. All of the Specialist's strands involve mathematics, although many are not primarily mathematics. The mathematics proper is woven into what I call the "mathematical landscape." This construct is similar to the "landscape of learning" that Fosnot and Dolk discuss in their *Young Mathematicians at Work* series [7]. In the mathematical landscape, I place primary emphasis on the connectivity. I picture the collection of mathematical concepts, ideas, results, and procedures being represented as hills and mountains. Most of the landscape is hidden from us using any one viewpoint; but, the landscape is connected by a complex web or network of pathways. There are often many paths connecting different mountaintops, and the journeys along different pathways provide the students with different understandings and knowledge. It is knowledge of this network of connections that provides individuals with their mathematical flexibility and power. The network reveals the mathematical relationships between topics. The mathematical representations and models that we use provide different viewpoints. Each representation offers a distinct view of a part of the mathematics. Mathematical relationships are formed by combining different viewpoints and mathematical representations.

I will illustrate my understanding of this landscape and the types of knowledge that a Specialist needs by peeking at this landscape through one multifaceted example, that of multiplication and area/array models. In this example, the Specialist's knowledge of the landscape should be highly connected and include knowledge of the following topics and links between them: multiplication; areas of rectangles including area and array models, and decomposition and recomposition of numbers; the distributive property and other laws of arithmetic; and, our base 10 number system.

1. A basic understanding of multiplication as repeated addition can be developed using either the area of rectangles or arrays of discrete objects, as illustrated below.



2. However, children must develop the spatial structure of rows and columns implicit in this model before the model can become the basis for significant generalization and abstraction. Case 19 in “Measuring Space in One, Two, and Three Dimensions” shows some of the difficulties students (third graders) may have drawing small arrays [9].
3. It is possible to use the area model in the development of the distributive property.



$$(a + b) \cdot (c + d) = ac + ad + bc + bd.$$

However, to do this, the students need to understand:

- multiplication, arrays;
 - the area model; and,
 - the fact that when a rectangle is decomposed into rectangular pieces, the area of the large rectangle equals the sums of the areas of the pieces.
4. In the area model, the factors in a product are lengths while the units for the product are square units of area. The units change! This naturally leads to the questions: Can the language of the area model effectively be adapted to discuss multiplication generally? What are the mathematical issues involved here? *Note that when points on the real line are used to model the real numbers, the product of numbers must be represented as a point on the line.*

5. The algorithm for multiplication of two-digit (and multi-digit) numbers rests upon the distributive property and is often explained using area or array models. To explain the computation 14×23 , a drawing similar to this (or base 10 manipulatives) is often used.

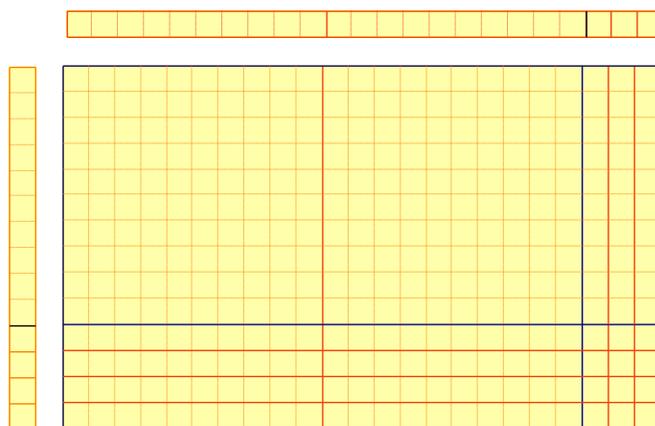


The pictorial representation illustrates the distributive property and shows that

$$14 \times 23 = (10 + 4) \times (20 + 3) = 10 \times 20 + 10 \times 3 + 4 \times 20 + 4 \times 3.$$

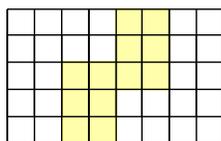
The naturalness of the area model seems to largely disappear when multiplying numbers with three or more digits because we cannot effectively draw accurate representations. One would hope that Specialists have encountered and worked through this issue. This next step of multiplying three-digit numbers seems to lead to significant abstraction. Substantial mathematical knowledge for teaching seems to be required here.

6. The problem of units, referred to in #4 above, reappears when base 10 materials are used to multiply decimals. For example, in the problem 1.4×2.3 , base 10 materials are sometimes used in the following manner:



In the frame on the outside, the rods are often incorrectly described as designating units while the small squares are said to designate tenths. In the array, the large squares are units, the rods are tenths, and the small squares are hundredths. The mathematical issue here is that, in the frame, 2-dimensional pieces are used to measure lengths and these same pieces appear in the rectangle as units of area. Serious confusion can result at this point and serious misunderstandings will likely result in the minds of our students if these mathematical errors are not addressed.

7. When arithmetic is extended to negative numbers, the array model is not an area model and, if used to include discussion of $(a + b) \times (c + d)$ where the terms may be either positive or negative, the model must be extended to become a signed area model. In my experience, this fact is almost never addressed.
8. Arrays also appear in work on fractions, decimals, and percents. The following problem where multiple solutions are sought is typical. What fraction, decimal, and percent of the large rectangle is shaded?



The Program's Courses

This multiplication example displays a significant amount of mathematical knowledge that one would hope Specialists possess. The topics which were mentioned are not discussed in most ordinary mathematics classes. Moreover, it seems to me that the understanding needed with each of these numbered items is typically something that will not be directly transmitted in a lecture, but requires thoughtful reflection and discussion by the learner. The Mathematics Specialist courses provide the participants with constant opportunities to reflect on and discuss such matters. Students in these courses must constantly explain their reasoning concerning problems, concepts, and solutions. They must react to solutions from other students. They read many case studies of student work from real classrooms (typically from *Developing Mathematical Ideas*) and they are expected to react to the student work, try to discern what understanding the students exhibit, and suggest appropriate pedagogical next steps [9].

The program was designed specifically to prepare teachers to serve as Mathematics Specialists in elementary schools. The development was done by teams of mathematicians and mathematics educators from the higher education partners and school mathematics faculty and supervisors. As would be expected, differences of opinion as to what mathematics is needed by Specialists occurred frequently, but everyone's voice was heard and consensus compromises were reached. The curriculum that emerged represents a broad consensus within the development teams on what Mathematics Specialists need to perform their jobs. The original disagreements did not, however, disappear and they continue to resurface seven years into the project. For example, active discussions persist on whether it is best to go deeper or to cover more material in *Numbers and Operations*.

The course, *Numbers and Operations (N&O)*, is a prerequisite for all other courses in the program. It closely follows the *Developing Mathematical Ideas (DMI)* numbers and operations materials, but this is supplemented by adding problems and additional work on topics, such as arithmetic with different bases [9]. This course sets the tone for all the mathematics courses where students are pushed to question, explain, and understand. Throughout the program, the standard the faculty and students are held to is that everyone must understand both how to solve problems and how to justify their solutions. A successful feature of the program is that students do not pretend they understand things which they do not.

The *N&O* course primarily treats whole number and fraction arithmetic, but the project staff recognized early on that additional work was needed on fractions, rational numbers, and proportional reasoning. There are no *DMI* materials appropriate for this course and several texts (Lamon, Fosnot and Dolk, and Smith, Silver, and Stein) are used and supplemented with additional activities [5, 7, 11]. This course develops a deep understanding in this strand—a strand dominating a large fraction of the middle school curriculum.

The course, *Rational Numbers and Proportional Reasoning (RN&PR)*, is followed by the course *Algebra and Functions (A&F)* that is primarily based on the *DMI* algebra materials. This course stresses early algebraic thinking, generalization, the development of the laws of arithmetic for the integers (and to a far lesser degree the rational numbers), functions, and symbolic algebraic arguments. Because their initial understanding is often limited, many of the students do not progress far in developing algebraic reasoning. An illustrative example showing this limitation is that, after completing this course, not all students who took a geometry course were able to find the length of an edge of a square of known area without assistance. Quadratic functions had been introduced (not treated extensively), but these students' ownership of the function and inverse function concepts was still very limited.

The course entitled *Probability and Statistics* uses the *DMI* text on data and a variety of materials on probability, including especially the NCTM *Navigations* text [9, 12]. The primary emphasis here is the development and use of the elementary tools of descriptive statistics, together with an introduction to probabilistic reasoning that develops such concepts as events, sample spaces, repeated trials, and independence. I judge the part of the course focusing on data to be quite successful. I have found the probability piece, especially that part dealing with conditional probability, to be highly challenging for many of the students. We have no research to document this, but it is my opinion that the largest obstacle to learning this material well is the students' limited fluency in and ownership of proportional reasoning.

Finally, *Geometry and Measurement (G&M)* covers the K-8 geometry and measurement topics with a strong emphasis on measurement in dimensions one, two, and three. The *DMI* geometry and measurement materials are used for approximately half of the course [9]. Activities from other sources, especially the Virginia Department of Education professional development materials and the unpublished text of Pitt, Timmerman and Wall, extend the course well beyond the limits of the *DMI* course [13, 14]. The standard K-8 area and volume formulas are all

discussed and derived (or given intuitive justifications). The Pythagorean Theorem, similarity, congruence and transformational geometry are all explored. The van Hiele model for how children learn geometry is discussed and a strong emphasis is given through developmentally appropriate student-centered activities. This is an area of real weakness for most of our Specialist students.

My concluding remarks for this section are:

- The program is intended to prepare K-5 Specialists. The operative interpretation here is that K-6 mathematics is covered in depth. My judgment is that, for the majority of the students, the program meets this goal well. The mathematics of grades 7 and 8 is discussed in these classes, but is not treated with the same depth as the K-6 curriculum.
- The program includes much discussion of MKT and it attempts to develop a solid familiarity with the K-6 mathematical landscape. Typically, graduates leave being well prepared to serve as K-5 Specialists. However, only those graduates who entered the program with a strong mathematical preparation for teaching middle school mathematics are well prepared to serve as 6-8 Specialists.

Mathematical Aptitudes and Abilities

In this final section, I offer a few observations and personal thoughts concerning the mathematical abilities and aptitudes that are needed by highly successful Mathematics Specialists. They are based on my extensive experiences in the program. I have been involved in designing and teaching all of the mathematics courses, and I have directed many of the final practicum projects of University of Virginia (UVA) graduates. In the practicum projects, students are asked to research, design, and implement a project in which they practice the work of a Mathematics Specialist. The projects provide the faculty with excellent opportunities to assess the students' potential as Mathematics Specialists. I have also been engaged in the admission process of more than 150 applicants to UVA's program. This has provided me with the opportunity to develop informed opinions about what this population looks like on paper. The combination of all of these experiences has allowed me to form opinions on the mathematical aptitudes, abilities, and skills that I would like Specialists to possess, and on the impact our program has had on individual teachers.

Beneficial Impact—The program has had a beneficial impact on every teacher who has completed it. I believe they are all better teachers than they were when they began the program.

Some of them are truly outstanding. The changes are the result of their new knowledge of mathematics and mathematical knowledge for teaching, as well as changes in their beliefs on teaching practice. In most cases, this has been dramatic and it began with the first course, *Numbers and Operations*.

Mathematics Specialist Position—The position of the Mathematics Specialist is very complex and demanding. In addition to the mathematics qualifications that I have written about, it requires the personal skills to work productively with students, teachers, administrators, and parents. A knowledgeable Specialist without these skills and personality traits may be an excellent teacher and a bad Specialist. They can easily damage the quality of instruction in a school, and administrators must pay close attention to these matters when selecting candidates to be Specialists. The importance of these issues can scarcely be overemphasized.

Profound Understanding of Fundamental Mathematics—The heart of a Mathematics Specialist's job centers on improving mathematics teaching and learning in the schools. Very often, this work will rest on their "profound understanding of fundamental mathematics," mathematical issues that are not understood by other teachers in the school. This role regularly requires a technical knowledge of mathematics, an intimate familiarity with the landscape of school mathematics, and a significant amount of mathematical flexibility. My experience has convinced me that not all elementary teachers can rise to the required level and that it is imperative that we do not endorse teachers below this level.

Because graduation from the state approved degree programs leads to an endorsement as a Mathematics Specialist, it is critical that these programs exercise standards that limit the number of unqualified graduates. I urge my colleagues to continue to investigate and discuss this issue. In my work with UVA students, I have gained valuable insights relevant to this situation.

- Successful completion of some courses with the title *Mathematics for Elementary Teachers* does not guarantee that the student has the abilities that I am advocating.
- Knowledge of college algebra and precalculus is not a prerequisite for Mathematics Specialists, but successful completion of such courses typically indicates possession of the sought after abilities.
- Standardized examinations, such as the GRE quantitative examination, are not precise tools for evaluating mathematical ability, but students with GREQ scores of

400 and below have typically struggled in our program for mathematical reasons. Students with GREQ scores above 500 have not struggled for mathematical reasons. When students take the GRE multiple times, their scores may vary significantly and when students prepare for this examination, their scores can rise dramatically. I believe that an appropriate cut score on the GREQ test lies somewhere between 400 and 500. Due to the variability of the scores, applicants in this range, but below the cut score, should be encouraged to take the test again.

Appendix A

Author's Background and Involvement in the VMSP

I grew up in the 1940s and 1950s in rural northern Idaho. This setting provided me a nearly ideal constructivist, hands-on environment to learn the mathematics and physical science of elementary and middle school; an environment where all problems came with a context that was compelling to me. The resulting childhood experiences started me on my way to becoming a mathematician (now retired) and lay the groundwork for my philosophy of mathematics education; a philosophy emphasizing enquiry, activities, and problem solving.

My path to becoming a (pure) mathematics professor included the standard bachelor's, master's, and doctorate degrees (including an M.S. in Biometry from Catholic University in 1964). I joined the mathematics faculty at the University of Virginia in 1970 and by the mid-1980s, I had become involved in working with mathematics teachers and schools. In 2000, I and a few school partners began work on conceptualizing a master's program for Mathematics Specialists, work which eventually became the Virginia Mathematics Specialist Project. When this partnership expanded and funding was received to develop a formal program, I led the project's development of its mathematics curriculum. I served on the development teams for each of our five basic mathematics courses and then taught each of these courses. In the years 2007-2009, I was the advisor for seventy-five students who graduated from the UVA Mathematics Specialist M.Ed. program.

Eight of the courses in the program are shared by UVA and our partner IHE. They were developed by teams consisting of school mathematics teachers and mathematics administrators, mathematicians, and mathematics educators. The curriculum decisions that were made reflected the committee's understanding of the Mathematics Specialist position and the knowledge and skills, including mathematical, pedagogical, and leadership, that the Specialists need to perform their jobs. My interactions with the development teams and with the students who have completed the program have shaped most of the opinions expressed here.

References

- [1] *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 2000.
- [2] *The Mathematical Education of Teachers*, Conference Board of the Mathematical Sciences, The American Mathematical Society, Providence, RI, 2001.
- [3] L. Ma, *Knowing and Teaching Elementary Mathematics*, Lawrence Erlbaum Associates, Mahwah, NJ, 1999.
- [4] *Adding It Up: Helping Children Learn*, National Research Council of Mathematics, Washington, DC, 2001.
- [5] A.H. Schoenfield (ed.), *Assessing Mathematical Proficiency*, Cambridge University Press, New York, NY, 2007.
- [6] D.L. Ball, H.C. Hill, and H. Bass, "Knowing Mathematics for Teaching: Who Knows Mathematics Well Enough to Teach Third Grade and How Can We Decide?" *American Educator*, **29**(3) (2005) 14-46.
- [7] C.T. Fosnot and M. Dolk, *Young Mathematicians at Work: Constructing Fractions, Decimals, and Percents*, Heinemann, Portsmouth, NH, 2002.
- [8] H. Hill, B. Rowan, and D.L. Ball, "Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement," *American Education Research Journal*, **42**(2) (2005) 371-406.
- [9] D. Schifter, V. Bastable, and S.J. Russell, *Developing Mathematical Ideas: Casebooks and Facilitator's Guides*, Pearson Education, Upper Saddle River, NJ, 1999-2010.
- [10] M.S. Smith, E.A. Silver, and M.K. Stein, *Improving Instruction in Rational Numbers and Proportionality: Using Cases to Transform Mathematics Teaching and Learning*, Teachers College Press, New York, NY, 2005.
- [11] S.J. Lamon, *Teaching Fractions and Ratios for Understanding*, Lawrence Erlbaum Associates, Mahwah, NJ, 2005.
- [12] G.W. Bright, D. Frierson, J.E. Tarr, and C. Thomas, *Navigating Through Probability in Grades 6-8*, National Council of Teachers of Mathematics, Reston, VA, 2003.
- [13] L.D. Pitt, M.A. Timmerman, and C.E. Wall, *Geometry for Teachers of the Middle Grades*, Dept. of Mathematics, University of Virginia, Charlottesville, VA (unpublished notes, 1998).
- [14] *Geometry for Middle School Teachers, A Professional Development Program*, Virginia Department of Education, Richmond, VA, 2004; Internet:
http://www.doe.virginia.gov/instruction/middle_school/mathematics/geometry_model/mid_entire_document.pdf.

HOW TEACHERS LEARN: THE IMPACT OF CONTENT EXPECTATIONS ON LEARNING OUTCOMES

J. REYES

*Dept. of Mathematics, Norfolk State University
Norfolk, VA 23504*

During the past seven years, the mathematics community in Virginia has been developing and offering programs to prepare teachers to serve as Mathematics Specialists/coaches and to devise means to support these Specialists as they assume their roles in the schools. We have received substantial support from the National Science Foundation to develop and offer these programs and to conduct research on their impact.

Upon the completion of these projects in December, 2009, the primary investigators hosted the “What We Have Learned Symposium.” The goal was to provide an opportunity for various collaborators to share their findings and observations from their different vantage points and areas of responsibility within the program. The Symposium consisted of discussion panels that were crafted to extract the collective experiences of four distinct groups: mathematics supervisors involved in the aforementioned program, Mathematics Specialists who had participated, instructors of the leadership courses, and instructors of the content courses. As a member of the latter group, I was charged with answering the following question:

What did you learn about the abilities/interests/background/attitudes/expectations of the teachers in the preparation program?

In forming an answer to this question, I gravitated toward the “expectations” part of the statement and began to reflect on my first impressions of the participants in the *Probability and Statistics* courses that I taught. I should first say that, as a college professor of six years, my experience with K-12 teachers has been limited to my participation in this project. Moreover, outside of undergraduate instruction, this was my first opportunity to serve math teachers in the capacity of primary content instructor. In doing so, I quickly learned that, at least initially, many participants in the program had seemingly low expectations for learning new material from the content courses of the program. This may seem like an odd assessment at first. However, it is necessary to keep in mind that: 1) the nature of the program is to recruit and prepare some of the most talented and motivated K-12 teachers; and, 2) the course content is K-8 based. It is then

more understandable that many participants were not lured by the opportunity to learn new content. Rather, it is more likely that many were primarily lured to the program by the opportunity to either gain licensure as a Mathematics Specialist or to obtain a master's degree. In other words, my presumption is that participants began the program with a high level of confidence in their knowledge content, whether justified or not. The reason this is significant is that it can conceivably make the learning environment quite different than the typical college course setting where students begin most courses with *no presumption* of understanding all (or even most) of the material that is to be presented.

Meet Jane Doe

Consider the following scenario as an example of the challenges that an MSP participant may face with respect to having a shift in learning expectations. Jane Doe has been selected to participate in an MSP program by virtue of her recognized excellence in the field of teaching and coaching other mathematics instructors. To her, the program has been presented as a mix of content and leadership courses that “include new content, focus on developing content across multiple grade levels, and seek to develop teachers’ pedagogical content knowledge for teaching.” Jane gleefully agrees to participate in the program. However, being unofficially labeled as one of the top teachers in her district, she has little or no expectation of gaining new *content* knowledge from this program. Instead, she focuses on the latter parts of the program description, and figures that the program will simply deepen her ability to convey the material to her colleagues once she is hired as a Mathematics Specialist. Let us now fast-forward to the third day of her first content class. Here sits Jane Doe, in the front of the class, realizing for the fourth time in three days that her knowledge of the concepts she *thought* she knew at a mastery level (and had taught for four years to students and colleagues alike) was shallow, at best. As the instructor continues the lesson and conversations ensue throughout the classroom, Jane sits silently and thinks to herself, “Wow. I never knew. How many times have I taught this concept incorrectly?” While the class pushes forward with the lesson, intertwining the concepts just presented with methods and thoughts on *how to teach the concepts to teachers and students alike*, Jane has shut down and will most likely be left behind. When and if she catches up to the rest of the class, it is conceivable that her focus will now be limited to trying to grasp the mechanics involved in the lesson and figuring out how the new ideas mesh with her prior knowledge.

Significant research has been done regarding how a classroom teacher’s expectations of student achievement affect (and essentially mold) the learning experience for the students involved. However, in this case, we are considering the expectation of the participant’s own learning; and more specifically, how the possible disconnect between expectations and reality can affect the learning outcomes. In short, Jane Doe’s hypothetical experience raises the question:

How does the need to learn (or in most cases, unlearn) content material affect the overall learning experience of a MSP participant who may not have had such expectations prior to starting the program?

Simply put, my hypothesis is that encountering this disconnect between “what I know” and “what I thought I knew” can be hindering for the remaining parts of the learning process. Initially, it often takes time for the participants to properly redefine their definition of “knowing” the mathematics. A mindset that I’ve typically encountered can be best summarized by the statement, “I can do it, but I don’t know it” which refers to most participants’ ability to complete the procedural portion of the mathematics without an adequate understanding of why, how, or even when the given procedures are applicable. If the course is successful, then what they eventually learn is that such an understanding of the mathematics is not a complementary aspect to what they would call content knowledge, but in fact *is* the content knowledge itself. It is this revelation that often shines light on the participants’ content knowledge deficit, and allows them to successfully move forward in the learning process. However, I have found that for some, recognizing the need to push past the doing to the knowing can be a long and painful process, to the point where it impedes their ability to embrace the learning experience and absorb the full value of the lessons at hand. This is what I term the “Jane Doe Experience.”

Redefining Content Knowledge: The Histogram Example

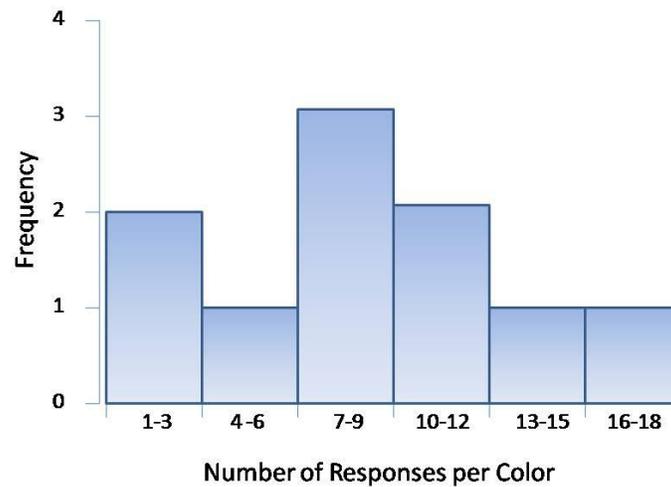
Consider the following example which, at least in its general form, took place during the presentation of an assignment during *Probability and Statistics*. A group of three participants were given the task of creating an experiment, collecting data, and choosing a graphical representation to display the data. To complete the first part of the assignment, the group chose to address the question, “What is your favorite color?” There was a fallacy in this part alone, as the data resulting from this query would not result in a continuous (versus discrete) data set as was required for this particular assignment. Nonetheless, the group collected the data (see Table 1).

Table 1
Results of Group Data Collection

Color	# of responses
Black	7
Blue	18
Brown	2
Gray	8
Green	10
Pink	7
Purple	10
Red	15
White	5
Yellow	1

To complete the assignment, the group chose to work with a histogram, and displayed their data (see Table 2).

Table 2
Color Histogram



The trouble was, at a glance, few participants could see the fallacy of this setup. On one hand, the group that presented this display clearly knew the mechanics of creating a histogram as it is presented in any mathematics textbook. That is, they understood the things that may have

been considered histogram “content knowledge” prior to the course. Moreover, one of the participants even mentioned the vague resemblance to a Normal distribution which did suggest some level of understanding of the usefulness of such graphs. However, what was lacking on both accounts was the understanding of the parameters in which a histogram can be applied, and how to garner a proper interpretation of the graph. These are two points that are arguably far more important than the procedural aspect of the lesson. To get the participants to see the fallacy of applying this graph type to this data set, the instructional team was careful to pose questions that did not suggest that the usage of this graph was incorrect. Rather, we framed the questions with the hope that the participants would reach this conclusion on their own based on their inability to properly interpret the graph. In the end, we were largely unsuccessful, even though no one could successfully answer the question, “what does this graph tell us about favorite colors?” (to which the answer is “essentially nothing”). Few could pinpoint the reason why the question (and lack of an answer) was important. That is, it was not made evident that this was the wrong choice of graph for this scenario. Moreover, as we pushed forward with the lesson and discussed the context in which a histogram is applicable and useful, and the resulting interpretations—evidence of the “Jane Doe Experience” began to materialize.

Conversations with Three Mathematics Specialists

With the Jane Doe Experience, the Histogram Example, and my initial hypothesis in mind, I followed up the Symposium with interviews of three MSP participants. The hope was to gain further insight into the expectations of participants with respect to learning, and the subsequent effect (if any) of those expectations. The manner in which the interviews were conducted was by no means scientific in nature. They were conducted in the form of a conversation, during which the participants were asked questions with the hope of leading them toward this subject without biasing their responses. At the end of the interview, they were then presented with the premise of the interview: namely, the italicized question in the section entitled “Meet Jane Doe.” They were then blatantly asked to agree or disagree with the hypothesis. What follows is a brief description of each interviewee’s teaching background, followed by the transcription of some of the responses collected, and a suggested model for teacher learning. Lastly, please note that the three participants were not all interviewed together, and therefore the sequence of excerpts presented here should not be construed as the exact sequence of the conversations that took place.

Throughout this article, I will refer to the three interviewees as Participants 1, 2, and 3; or simply, P1, P2, and P3. Of the three, P1 was the most experienced, having taught for fourteen years. She is preK-4 certified, and has taught third grade for eleven years. With six years of experience and preK-3 certification, P2 has been a classroom teacher for first, second, and third

grades. Participant 3 has also taught for six years, but previously spent an additional four years as a substitute teacher. As a full-time teacher, she has taught fifth grade exclusively and is preK-6 certified.

In each case, the first part of the conversation was centered on determining how each participant was recruited for the program. The goal was to determine when they were told about the content involved in the courses, and how the delivery of this information began to shape their expectations of the program. Here are their responses.

- P1: The Senior Math Coordinator for my school district told me about the program. I had just become a Math Resource Teacher. She mentioned that it would be three summers and a few leadership courses, but that's about all I knew initially.

Participants 2 and 3 had very similar accounts. Each was told that the program would last three years and that a certification or licensure (which was not yet approved) would be involved. Participant 2 was quick to point out that a list of courses was not given, while P3 mentioned that she knew little about what would be involved besides the opportunity to be a Math Resource Teacher at the end of the program. From here, the question was asked, "So when did you find out more about the courses that you would be taking?"

- P3: I didn't find out anything else until I walked in the door [of the first class]. We did take a pre-test [prior to the start of the program], but for some reason I didn't think that the pre-test related much at all to the courses that I would be taking.
- P2: I remember that we went to take a pre-test downtown. Dr. McNeil (one of the Norfolk State University professors) was there. At that point, we learned more about the program and we also took the pre-test for the first course and were told to take the GRE. [In terms of the pre-test], I didn't think much of it. There was a lot of stuff on there that I didn't know, but I just thought to myself, "I must have forgotten all of this"; but, I wasn't overwhelmed. I figured it would all come back once I got into the class.
- P1: At some point before the classes started, I found out that [the program] was about K-8 certification. And I knew the subjects [that would be covered]. I remember thinking, "Okay, I know those." I was more curious as to how they were going to show me this "new math" and how I was going to adapt to it and be able to relay it to my students. I was told that it wouldn't be the usual procedural stuff, but I was skeptical. At that time, there was little thought about the content.

The comments of Participants 1 and 2 are consistent with the premise that, for at least some participants in the program, the focus was on pedagogy and the level of expectations for learning new content was low. In fact, prior to the program starting, all three interviewees seemed squarely focused on the extent that the courses would build upon pedagogy and leadership skills. Little thought was put toward the extent that they might be challenged by the content involved. Along those lines, Participant 1 also added that she had already taken courses at the master's level and had simply thought, "How hard can it be?"

With this established, each conversation was then shifted toward the interviewee's personal assessment of what was learned versus the aforementioned expectations.

- P2: *Numbers and Operations* was the one class [in which I felt] I knew the material beforehand. All of the other classes were me learning the math [for myself]. I struggled more with the content of the other classes. I think it was the same thing for [two other participants]. With this class, I knew the content and [so I was able to] go deeper with it. But with the others, it was more figuring out the meaning of the math. For me as a learner versus as a teacher, it took an adjustment. First I had to realize that I'm not stupid. [...] It helped that other people struggled, too, which meant that I wasn't the only one in this boat.
- P3: [About the course *Numbers and Operations*] WOW... it was eye opening. I only taught the fifth grade, so for me it was so great to see the foundation. They weren't teaching me "how to"; they were teaching me "why." In terms of learning the material [as a cohort], it was key that we felt so comfortable with each other. There was a good disequilibrium among all of us, and so I found real value in the cohort. We came together nicely.
- P1: Once I got into the class, it was all very different. It started with *Numbers and Operations*. It was not as bad as I thought. There were procedural [methods] involved, and also things like being able to decompose and recompose. As the program progressed, the most difficult class was *Probability and Statistics*. [To me], everything was pure procedure before that point. When I got to *Algebra* which was my strongest point, I was comfortable. [...] In each class, I think that my level of adaptation was based on how strong I was in that particular area.

In their own way, each of the three participants proceeded to elaborate on the extent to which the content courses challenged them. The common sentiment can be summarized simply using the word "unexpected," as all three of the participants seemed blindsided by the extent to

which they were challenged. Thus, having established that there was indeed a wide disconnect between expectations and reality, we finally turned our attention to the crux of the discussion: “How did this disconnect affect the ability to learn pedagogy, leadership skills, or the presented content itself?” More specifically, the participants were asked if learning so much new content hindered or helped the learning process with respect to learning the pedagogy and finding new ways to convey this material to students and colleagues. For Participant 1, the answer was mixed.

P1: At first, it was hindering. I became defensive. For me, being certified for K-5, I came in thinking about learning new ways to teach the concepts. [I asked myself] am I going to be able to effectively convey this to my students and colleagues? I didn't expect to struggle. [In terms of the math], I knew that there are tricks that work, but I never thought about why the tricks worked. In the *Rational Numbers* class, I remember learning different ways to solve a problem, but [they told us] you cannot use procedural [methods]. I could not think of another way. There were times where I shut down. There were also times when I embraced it, but not often. If I could take a class over, it would be that one.

[...] But looking back now, I think that learning the content on such a new level improved my ability to absorb the pedagogical procedures conveyed because in my head I was learning along with the student. [And as I reflected on my prior knowledge] I felt cheated with respect to how I was taught. But I used this as motivation to learn and move forward despite how difficult the material became. [Feeling cheated] was the first step necessary for me to be willing to abandon my old ways of thinking.

While her initial response was the quintessential Jane Doe Experience, her reflection at the end suggested that she was able to overcome not only these difficulties, but was able to ultimately use them to her advantage. Participant 2 didn't feel as lucky.

P2: For [the] *Rational Numbers* [course], I didn't get the satisfaction that I got from other classes. All of my most frustrating moments [happened in that class]. I didn't know what to ask [...] and so I participated less. I never got past learning for myself.

As it turns out, in my assessment of our conversation, it seemed that Participant 3 did not have a Jane Doe Experience at all. While she came across material that was unexpectedly challenging, it never seemed to affect her ability to move forward. We will further discuss the thoughts of Participant 3 on encountering new content as we close in the following section.

The Focus Model

The interviews in the previous section were focused on determining the ratio of time that MSP participants spent focused on learning new content versus learning about methods of instruction as they pertain to the content. Two of the interviewees confirmed that this ratio being higher than anticipated negatively affected their overall learning experience at some point in time. To help explain this phenomenon, the participants and I came up with what I call the “Focus Meter” model of teacher learning (see Figure 1).

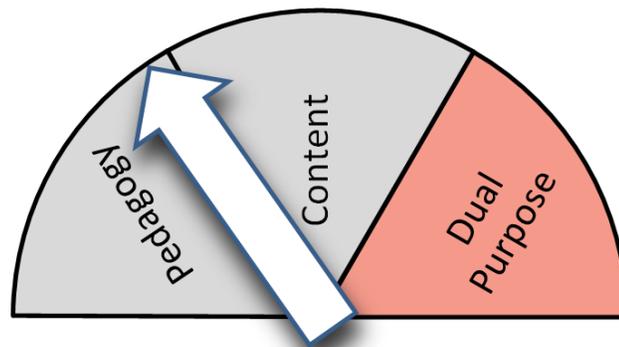


Figure 1. The “Focus Meter” model of teacher learning.

The diagram models the mindset of a teacher during the course of a program such as the MSP. The indicator arrow points toward the participant’s intended point of focus in a course, *as they see it*. While a participant can certainly experience both improved pedagogy *and* improved content knowledge simultaneously, it is reasonable to think that most participants will expect to focus their attention primarily on one of these two areas. Which area they choose may be based on their background and how the course is advertised. Accordingly, there are three sections in the Focus Model dial, each based on the type of expected learning outcome: improved pedagogy (S1), increased content knowledge (S2), or a dual purpose approach that allows the learner to serve both ends simultaneously (S3).

It seems that although some learners are restricted to the first two sections, it is still easy for them to switch from S1 to S2 as needed. For example, Participant 2 presented herself as more of a linear thinker. That is to say that she had to *fully* absorb and understand the content before

moving on to strategies to convey the material. Presumably, these are the participants that are most susceptible to the Jane Doe Experience. Participants 1 and 2 were alike in that both seemed unlikely to achieve S3 status. However, Participant 2 did seem able to move her indicator arrow back and forth between S1 and S2 within the same context. For teachers similar in mindset to Participant 1, this back and forth between S1 and S2 is achievable within a single lesson, but the participant's personality may dictate whether this is an effortless action or whether it takes a conscious effort to move the needle from one section to another. Either way, the good news is that it is possible, and not necessarily a painful process for the learner.

Section 3 of the Focus Model dial is the ideal situation in which the participant can recognize the need to absorb new content while simultaneously considering strategies to subsequently convey this deeper knowledge to other teachers and students. This section of the dial is not an area that is easily attained by all learners. Participant 3, however, seemed to reside quite comfortably in Section 3. During her interview, she discussed the value of a high "new content" to "pedagogy" ratio as it pertains to achieving the desired learning outcomes.

P3: I had a tendency to enter each class with an "I can do this" mentality. And even though that wasn't always the case, I don't think that learning new material ever really threw me for a loop. [Even in challenging situations, I remained] pretty comfortable switching from "learning for me" versus "learning for my students." What I realized is that when you learn for yourself, there's this overlap because you can learn for your students at the same time and figure out how *they* are learning.

For such learners, the end result is arguably richer. They are able to experience the potential pitfalls, typical questions, and "aha moments" for themselves—all while *in the midst* of considering how to convey the material to a third party. This approach can promote better retention of lessons learned and thus enhance their ability to coach. However, the inability to reside in Section 3 does not spell doom for the other types of learners. While the details of their experiences may have been different, we are reminded that all three of the interviewed participants essentially started and ended at the same place. The program began with each participant having little expectation of changing their content knowledge set; and, during the course of the program, the participants certainly differed in how they handled the obvious disconnect between their expectations and reality. However, in the end, experiencing first-hand the learning curve of new content knowledge clearly benefited *each of them* as they were able to embrace the pedagogical aspects of the lesson on a more personal level. For some, this embrace was simultaneous to learning for themselves. For others, it was subsequent. For all, however, the transformation of thought processes could be deemed a success as the ability to unlearn, re-learn,

and think more deeply about content knowledge and the corresponding pedagogy that was still achieved across the board.

From here the question that lies ahead is, “What can be done to better promote ‘Section 3’ experiences within teacher learning environments?” With five more years of our MSP program ahead of us, we will hopefully have plenty of opportunities to uncover and begin to implement the answers to such questions.

EARLY ALGEBRA AND MATHEMATICS SPECIALISTS

M.K. MURRAY

*University of Virginia Mathematics Outreach Office
School of Continuing and Professional Studies
Charlottesville, VA 22904*

Abstract

This paper discusses early algebra as it relates to the Mathematics Specialist program. Early algebra is described based on research and readings from the body of literature focused on early algebra. Reasons why early algebra should be emphasized in elementary school mathematics are discussed, followed by a description of the role elementary school Mathematics Specialists must play if schools are to begin to focus on early algebraic instruction. Finally, some suggestions are made for ways the Mathematics Specialist program might encourage more explicitly an early algebraic approach to elementary school mathematics.

Introduction

I have been an instructor many times for the courses taught for Mathematic Specialists in Virginia. Each time I have taught one of these courses, I have deepened my own understanding of the mathematics that elementary children are capable of understanding, as well as of ways in which children come to express these understandings. However, it wasn't until I had taught all three of the courses that make up what I think of as the "Numbers" sequence (*Numbers and Operations*; *Rational Numbers and Proportional Reasoning*; and, *Patterns, Functions and Algebra*) that I began to appreciate the connectedness and complexities of these courses. Furthermore, I had not really understood how these courses work together to support a curriculum focused on early algebraic reasoning. My work with these courses has led to my interest in early algebra and the research in the field. In this paper, I want to describe a little of what is meant by early algebra, based on research and readings from the body of literature focused on early algebra. I will discuss reasons why early algebra should be emphasized in elementary school mathematics. Next, I will look at the role elementary school Mathematics Specialists must play if schools are to begin to focus on early algebraic instruction. Finally, I will make some suggestions for ways I see the Mathematics Specialist program might encourage more explicitly an early algebraic approach to elementary school mathematics.

Early Algebra: What Is It?

The *Principles and Standards for School Mathematics* describes six content standards for grades K-12 [1]. The Algebra Standard envisions students who:

- Understand patterns, relations, and functions;
- Represent and analyze mathematical situations and structures using algebraic symbols;
- Use mathematical models to represent and understand quantitative relationships; and,
- Analyze change in various contexts.

It is important to realize that this Standard spans the elementary and secondary grades. Algebra is a body of knowledge that students learn over a long span of time, beginning in the early grades. Indeed, algebra is not separate from the arithmetic studied in the elementary grades; rather, algebra and arithmetic are integrally connected.

It is also important to understand that early algebra is not what we understand as high school algebra taught in earlier grades. Most researchers echo Carpenter and Levi who claim the goal of early algebra is to develop algebraic thinking [2]. They, like other researchers in the field, conceive of algebraic reasoning as the building, expression, and justification of generalizations, representing mathematical ideas with symbols, and using those symbols to represent and solve problems [3-8]. The algebraic reasoning most appropriate for elementary school that is the focus of these researchers' work typically falls into one of two subcategories: *generalized arithmetic* and *functions*.

Generalized Arithmetic—This term refers to the reasoning that occurs as students recognize patterns that emerge during their study of the four basic operations, and to the claims they make and later justify, and eventually express with symbolic notation. For example, a student solving the problem $37 + 28$ may take 3 from the 28 and add it to 37; the resulting problem becomes $40 + 25$. At first, the student may state a generalization of what he notices as with words: “When you take an amount from one addend and add the same amount to the other addend, you still get the same total when you add them together.” This serves as the basis for the symbolic expression of the relationship, $(a+b) = (a+c) + (b-c)$.

Functions—This term refers to the generalization of numeric patterns. Such patterns often arise from contextual situations, and may be represented with pictures, number lines, function tables,

symbolic notation, and graphs. For example, six pennies are added to a jar every day and the children analyze the growth.

An essential ingredient of early algebraic instruction is the focus on student reasoning and the discourse that allows students to identify connections among concepts, and then build on these connections to form generalizations. This discourse does not occur naturally, but rather is the result of a well articulated plan, developed by a teacher who herself understands the underlying algebraic aspects of the content. So early algebra is not just appropriate content, but also requires effective pedagogy to bring the deep meaning of the content to the surface.

Why Emphasize Algebra in Elementary Grades?

There are several reasons why an emphasis on early algebra in elementary grades is warranted. First, there is a call for early algebra on both national and state levels. Nationally, there is an emphasis on having all students complete at least one algebra course before graduating from high school. The NCTM released a position paper claiming all students should have an opportunity to learn algebra; furthermore, students need opportunities to encounter algebraic ideas across the PreK-12 curriculum [9]. Statewide, Virginia students are required by the Virginia Department of Education to pass at least three mathematics courses at or above the level of Algebra I in order to obtain a Standard Diploma [10]. The Virginia Department of Education's "Mathematics Standards of Learning" require students to explore algebraic concepts in grades K-6 [11]. Some examples of algebra content in these grades include: the formal exploration before sixth grade of the commutative, associative, and distributive properties; an understanding of equality and inequality by second grade; and, the ability to recognize and "describe a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms" by third grade [11].

Another reason to emphasize early algebra in the elementary schools focuses on issues of equity. The Equity Principle states, "All students need access each year to a coherent, challenging mathematics curriculum taught by competent and well-supported mathematics teachers" [1]. Schifter, et al. report that a focus on algebraic representations, generalizations, and connections supports students' computational fluency [6]. Furthermore, in the same article they provide evidence that working on developing algebraic reasoning supports the range of learners in a classroom. Less capable students begin to find the mathematics more accessible as they are offered more entry points; more capable students find the content associated with early algebra

“challenging and stimulating.” Thus, a curriculum grounded in early algebra offers greater opportunities for differentiation practices that are focused on substantial mathematical thinking.

A third argument for an emphasis on early algebra revolves around improving overall elementary mathematics curriculum. A curriculum focused on early algebra, with a constant eye on helping children build on past experiences to form generalizations that can be justified, will be much more coherent than a curriculum that “covers the Standards.” A curriculum tied together by algebraic concepts makes sense, and in fact might reduce what seems to be an overwhelming amount of material to learn by providing opportunities to teach more concepts simultaneously [12]. A simple case: understanding the commutative property reduces the number of basic facts one must learn by half. A less simple case: understanding how the distributive property is applied when multiplying whole numbers allows a student to apply the same process when multiplying mixed numbers. Another less simple case: approaching fact instruction through a functional lens creates opportunity for meaningful graphing experiences, tied to pattern exploration and tabular representations.

One aspect of the work on early algebra that seems so promising is that it does not require an entire reworking of the current elementary curriculum. Rather, as Carraher, et al. state, “existing content needs to be subtly transformed to bring out its algebraic character” [7]. Kaput refers to this as “algebrafying” the elementary school curriculum [3]. This “algebrafication” requires “acknowledging the several different aspects of algebra and their roots in younger children’s mathematical activity.”

Enter the Mathematics Specialists

Kaput and Blanton claim “elementary teachers are in the critical path to longitudinal algebra reform, yet they typically have little experience with the rich and connected activities of generalizing and formalizing” [13]. One predictable result of this lack of experience may be a lack of depth of understanding achieved by students, even those who are successful with the *Standards of Learning*. For example, consider two students who are asked to decide if $37 + 52 > 38 + 51$. Student 1, taught by a teacher without a deep understanding of algebraic concepts, will likely resort to simply adding both sides of the equation, obtaining the same answer, and claiming the statement to be false. This is true, but an opportunity has been missed to use what Carpenter, Franke, and Levi refer to as relational thinking [14]. Also, this student has not been given an opportunity to solve this problem in ways that provide initial experiences with commutative and associative properties. Student 2, taught by a teacher with a deep understanding of the concepts

and generalizations that can come from this problem, would likely solve this problem in a far different manner than Student 1. Student 2 might reason that 37 is one less than 38, but 52 is one more than 51, so the two sides are still even, using number sense and the relations between the numbers to arrive at a correct answer.

If elementary teachers lack the content and pedagogical knowledge necessary for providing the type of instruction focused on early algebraic reasoning, then clearly this is an area for their professional development. Several groups have reported their efforts in working with teachers as they begin to approach instruction of the elementary mathematics curriculum through an algebraic lens [15-17]. The approaches of these groups reflect the “algebrafication” strategy described by Blanton and Kaput [15]. This strategy is focused on classroom teacher change, approached along three avenues: 1) the “algebrafication” of instructional materials; 2) the support of students’ algebraic thinking; and, 3) the creation of a classroom culture and teaching practices supportive of algebraic reasoning.

Mathematics Specialists are in a critical position to provide sustained professional development focused on algebraic reasoning. In their daily work with teachers, Mathematics Specialists regularly work with teachers to plan daily lessons and overall curriculum, work that includes modification of existing instructional resources. In schools with Mathematics Specialists, teachers are becoming better adept at listening to and exploring student reasoning, and helping students build on their own reasoning. As a result of efforts on the part of Mathematics Specialists, more and more teachers afford students opportunities to explore and deeply engage in mathematical explorations, and classroom cultures are established that respect individual reasoning. So, the basic structures of “algebrafication” are in place as a result of Mathematics Specialists in schools.

Yet for “algebrafication” to occur, early algebraic reasoning needs to become a focus of the Mathematic Specialists’ work. Specialists need to provide opportunities for the teachers in their school to explore algebraic concepts for themselves in order to gain some depth of understanding of early algebra. As a Specialist works with teachers on lessons and curriculum, for example, the focus can be on underlying algebraic aspects of the concept in question, and how those aspects are brought to the forefront of discussions and developed into generalizations. Mathematics Specialists should work with teachers across the grade levels in their school to ensure that algebraic reasoning develops across concepts and from grade to grade, and that generalizations developed in one grade continue to be considered and reconceived or justified in

the next. Mathematics Specialists can help teachers recognize opportunities that arise to help children form generalizations, thus supporting students' algebraic reasoning while creating a community where that reasoning is expected and valued.

Explicitly Focusing Mathematics Specialists on Early Algebra, During the Program and Beyond

Much of the work done in the “Numbers” courses of the Mathematics Specialist program focuses on algebraic reasoning. One of the first activities prospective Mathematics Specialists enrolled in the *Numbers and Operations* course engage in requires them to solve a problem like $57 + 36$ using mental math. After a minute or so of reflection, participants share their strategies. Participants will propose a number of strategies, including: adding tens, then ones; changing $57 + 36$ to $60 + 33$ or $53 + 40$, then completing the work with these easier, benchmark numbers; and, starting at 57 and counting on (57, 67, 77, 87, 88, 89, 90, 91, 92, 93). The language in the activity includes words like decomposing and recombining; the concepts being developed underlie the commutative and associative properties of real numbers. Other work in this course continues to examine how children use number sense to develop meaningful approaches to the four operations; these approaches often rely on (yet unstated or formulated) properties of equality.

Algebraic reasoning is an essential component of the *Rational Numbers and Proportional Reasoning* course. Work with equivalent fractions, for instance, can be viewed through a functions lens. Examples of explorations teachers encounter include looking at similar rectangles, and examining the ratio of height to width with tables and through graphing. An arrangement of nested similar rectangles on the coordinate grid reveals that the diagonals of similar rectangles fall on the same line, connecting the table to a linear function and a discussion of slope. Multiplication of fractions is analyzed through an area model, but also as the result of an operator acting on a quantity; again, a function approach.

The course *Patterns, Functions, and Algebra*, in its name and content, is the course most obviously focused on algebraic thinking. In the first half of this course, the focus is on the generalization of patterns, developing skills necessary to describe patterns with symbols. Participants develop fluency with algebraic notation as they learn how the symbols represent the physical quantities and actions. Conjectures (e.g., an odd plus an odd equals even) are justified and proven to hold over fields of numbers first with models, then symbolically. Participants use models to justify laws of equality. In the second half of the course, activities explore various functions, with an emphasis on the connections between multiple representations. Work in this

course includes developing an understanding of how young children can develop an understanding of functions.

Clearly, opportunities to develop Mathematics Specialists' understanding of algebraic reasoning are available in the program courses. However, it is not clear that participants in these courses are aware of the algebraic nature of this work until they enroll in *Patterns, Functions, and Algebra*. As instructors, we miss opportunities to explicitly relate work in *Numbers and Operations* and *Rational Numbers and Proportional Reasoning* to algebra, and fail to explicitly highlight how algebra permeates the elementary curriculum. Just as a focus on early algebraic reasoning ties together the elementary curriculum, creating opportunities to teach more concepts in a connected manner and with richer understanding, a focus on algebraic reasoning could also serve to tie together "Numbers" courses in a more cohesive program.

How can the algebraic thread be made more explicit, in order to prepare Mathematics Specialists to think about early algebra in their own practice? First, some decision needs to be made as to the importance and relevance of algebraic reasoning as a unifying thread for these courses (and indeed, all content courses in the program.) If there is general agreement that algebraic reasoning should receive consistent, explicit focus, then instructional staff would benefit from professional development that highlights algebraic reasoning in the courses, and how the courses are related in this regard. This seems especially important for instructors who have not had the opportunity to teach all three of these courses, and to experience these connections themselves. The present Mathematics Specialist curriculum implicitly encourages algebraic reasoning from the onset; would it be even more powerful to encourage algebraic reasoning with more intent?

Mathematics Specialists also need support as they take on the work of implementing an early algebraic curriculum in their schools. This work should be focused on continuing to develop Mathematics Specialists' understanding of early algebra. Some of this work already occurs through conference sessions, some through local efforts. While it is not (currently) in the scope of the Mathematics Specialist program, continuing professional development focused on increasing endorsed Mathematics Specialists' knowledge of algebra could be considered in future initiatives.

Finally, a focus on algebraic instruction in elementary school is fairly new in the arena of mathematics education. Teaching number facts through a functions approach will look and feel

different to teachers, administrators, parents, and children; using patterns to learn these facts is also foreign to those who see this as a rote skill. Mathematics Specialists will need to advocate for this approach, and will need support in their advocacy. Mathematics educators involved in the Mathematics Specialist program need to work with administrations to develop an understanding and support for taking this approach to the elementary mathematics curriculum, because to be effective it will require time and effort in training staff and reworking curriculum.

Early algebra and algebraic reasoning is a relatively new area of research in the mathematics education literature. There is still a lot of research that needs to be conducted to determine how children learn to reason algebraically, and what this means for instructional practices and resources. If this research is best conducted in school settings, it follows that Mathematics Specialists should play a vital role, both as research subjects and researchers. To do so, they need to be prepared.

References

- [1] *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 2000.
- [2] T. Carpenter and L. Levi, *Developing Conceptions of Algebraic Reasoning in the Primary Grades*, National Center for Improving Student Learning and Achievement in Mathematics and Science, Madison, WI, 2000; Internet: <http://www.wisc.wcer.edu/ncisla>.
- [3] J. Kaput, "Transforming Algebra from an Engine of Inequity to an Engine of Mathematical Power by 'Algebrafying' the K-12 Curriculum," in *The Nature and Role of Algebra in the K-14 Curriculum: Proceedings of a National Symposium*, Washington, DC, 1998.
- [4] J. Kaput, "Teaching and Learning a New Algebra," in E. Fennema and T. Romberg (eds.), *Mathematics Classrooms that Promote Understanding*, Lawrence Erlbaum Associates, Mahwah, NJ, 1993.
- [5] J. Kaput, "What is Algebra? What is Algebraic Reasoning?" in J. Kaput, D. Carraher, and M. Blanton (eds.), *Algebra in the Early Grades*, Lawrence Erlbaum Associates/Taylor Francis Group, Mahwah, NJ, 2008.
- [6] D. Schifter, S.J. Russell, and V. Bastable, "Early Algebra to Reach the Range of Learners," *Teaching Children Mathematics*, **16**(4) (2009) 230-237.
- [7] D. Carraher, A. Schliemann, B. Brizuela, and D. Earnest, "Arithmetic and Algebra in Early Mathematics Education," *Journal for Research in Mathematics Education*, **37**(2) (2006) 87-115.

- [8] D. Carraher, A. Schliemann, and J. Schwartz “Early Algebra Is Not the Same as Algebra Early,” in J. Kaput, D. Carraher, and M. Blanton (eds.), *Algebra in the Early Grades*, Lawrence Erlbaum Associates/Taylor Francis Group, Mahwah, NJ, 2008.
- [9] “Algebra: What, When, and for Whom: A Position Paper of the National Council of Teachers of Mathematics,” National Council of Teachers of Mathematics, 2008; Internet: <http://www.nctm.org/about/content.aspx?id=16229>.
- [10] “Graduation Requirements,” Virginia Department of Education; Internet: <http://www.doe.virginia.gov/instruction/graduation/index.shtml>.
- [11] “Mathematics Standards of Learning, 2009 Adoption,” Virginia Department of Education; Internet: <http://www.doe.virginia.gov/instruction/graduation/index.shtml>.
- [12] J. Kaput, D.W. Carraher, and M.L. Blanton, “Skeptics’s Guide to Algebra in the Early Grades,” in J. Kaput, D. Carraher, and M. Blanton (eds.), *Algebra in the Early Grades*, Lawrence Erlbaum Associates/Taylor Francis Group, Mahwah, NJ, 2008.
- [13] J. Kaput and M. Blanton, “Algebrafying the Elementary Mathematics Experience,” in H. Chick, K. Stacey, J. Vincent, and J. Vincent (eds.), *Proceedings of the 12th ICMI Study Conference on the Future of the Teaching and Learning of Algebra*, **1** (2001) 344–351.
- [14] T. Carpenter, M. Franke, and L. Levi, *Thinking Algebraically: Integrating Arithmetic and Algebra in Elementary School*, Heinemann, Portsmouth, NH, 2003.
- [15] M. Blanton and J. Kaput, “Developing Elementary Teachers’ ‘Algebra Eyes and Ears,’” *Teaching Children Mathematics*, **10** (2) (2003) 70-77.
- [16] V. Jacobs, M. Franke, T. Carpenter, L. Levi, and D. Battey, “Professional Development Focused on Children’s Algebraic Reasoning in Elementary School,” *Journal for Research in Mathematics Education*, **18**(3) (2007) 258-288.
- [17] D. Schifter, V. Bastable, S.J. Russell, L. Seyforth, and M. Riddle, “Algebra in the K-5 Classroom: Learning Opportunities for Students and Teachers” in C.E. Greene and R. Rubenstein (eds.), *Algebra and Algebraic Thinking in School Mathematics: Seventieth Yearbook*, 2008.

GEOMETRY EXAMPLES ENCOUNTERED IN VARIOUS EVERYDAY EXPERIENCES

R.W. FARLEY

Professor Emeritus of Mathematics

Dept. of Mathematics and Applied Mathematics, Virginia Commonwealth University

Richmond, VA 23284-2014

Introduction

A student who is learning or relearning elementary geometry can benefit from applications that occur in everyday life. By explaining these applications, the study of geometry is not only enhanced, but the student is left with an appreciation for their relevance. Some of these applications can be used in a pedagogical content course. They range from the household to the construction industry to landscaping to hydraulics. All involve elementary geometry mainly involving areas and volumes.

Finding the Rectangle

Early in their study, Mathematics Specialist trainees learn that a necessary and sufficient condition for a parallelogram to be a rectangle is that the diagonals are equal. So, how is this interesting fact useful in daily work? One practical example is a method used by builders and involves ensuring that the foundation for a rectangular building, including many houses or rooms or parts thereof, is indeed rectangular. First, stakes with boards are installed on each corner. Then strings are tightly strung around the site in a pattern around nails on the corner boards (see Figure 1).

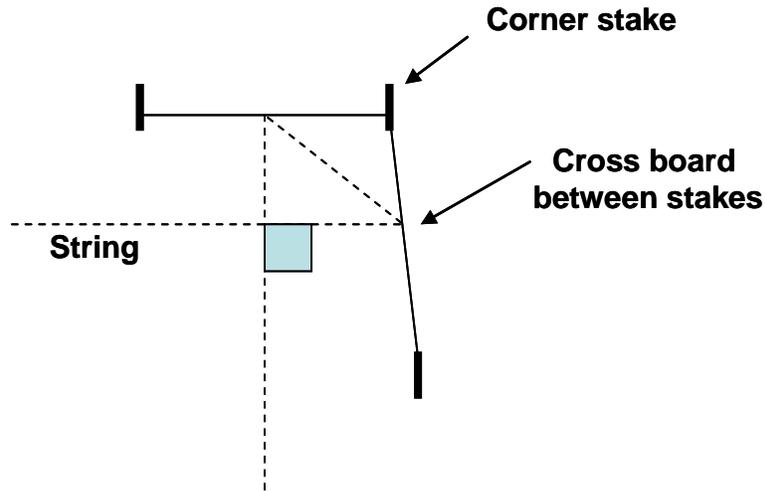


Figure 1. Aerial view of a foundation.

These string lines are used by the foundation bricklayers to plumb the necessary rectangular foundation. The problem is that using a square to measure the corner angles marked by the string is difficult to produce the necessary accuracy. So, the builder gets the string configuration reasonably close to producing the corner right angles, then measures diagonals for equality, and refines the string configuration until the diagonals are equal. This geometry has been used for decades by many builders. Students in several Mathematics Specialist geometry classes have enjoyed using an electronic range finder to measure outdoor “rectangular” regions.

“Would You Like Shag or Berber?”

A household application can be as simple as ordering the correct number of square yards of carpet to cover a 9' x 12' room. While students may experiment correctly using the measurement in feet to produce $9 \times 12 = 108$ square feet, they must then convert to square yards by recalling that $3 \times 3 = 9$ square feet which is one square yard. Then, they divide 108 by 9 and correctly obtain 12 square yards. Another method of calculation would be to convert the original measurements to 3 yards by 4 yards and easily multiplying to obtain 12 square yards.

“How Much Paint?!”

Another simple household application involves how many gallons of paint to buy in order to paint the four walls and ceiling of a rectangular room measuring $10 \times 14 \times 8'$ (high). Instructions on the gallon paint can state that normal coverage is 275 square feet. Of course, two

walls measure 14' x 8' each and the other two measure 10' x 8' each. Adding the 384 square feet of wall space to the $14' \times 10' = 140$ square feet of ceiling surface yields 524 square feet. Since the room has a 4' x 7' door in one wall which will not be painted, we can be comfortable with buying two gallons of paint.

Pouring the Footing

An additional building example involves the measurement in cubic yards of the amount of concrete to order to pour the “footing” on which the brick foundation will be laid for constructing a 60' x 120' ranch style house. The footing consists of pouring cement into a 16” wide trench so that the depth of the cement is uniformly 8” deep. Converting these measurements

into feet, the resulting calculation will be $\frac{16''}{12''} = \frac{4'}{3}$, $\frac{8''}{12''} = \frac{2'}{3}$ so that the volume of concrete in

the trench is $\frac{4'}{3} \times \frac{2'}{3} \times 360 = \frac{8}{9}$ square feet times 360 feet long = 320 cubic feet. Recalling that

a cubic yard is $3' \times 3' \times 3' = 27$ cubic feet, the resulting division, $320 \div 27$, yields slightly less than 12 cubic yards.

An interesting question results from our calculation. Since the depth and width of the poured concrete is constant for most normal building applications, can we simplify repeated calculations by finding a constant K (which we will call “the footing constant”) which can always be multiplied by a varying length to quickly produce the number of cubic yards of concrete needed? Students can decide whether to work with units of either feet or yards. One solution is

to notice that our calculation above involves $\frac{8}{9} \times 360' (\text{length}) \div 27$. So, if we use

$\frac{8}{9} \div 27 \times \text{length} = \frac{8}{243} \times \text{length}$, the result will be the cubic yards required. Thus, the constant

K is $\frac{8}{243} \approx .033$. So, for example, if we need to pour 260' of uniform footing, we need

$.033 \times 260 = 8.58$ cubic yards of concrete. Effectively, we will order 9 cubic yards of concrete.

The calculation requires the calculation $\frac{8}{9} \times 360' \div 27 = \frac{8}{9} \div 27 \times 360'$. Use number properties to verify this equality.

Circular Logic

Landscaping offers further opportunities to explore geometry. The problem: how many Belgian path stones 8" long will we need to form a circular mulch border of a 10' radius from the center of a tree which has a 12' circumference at its base?

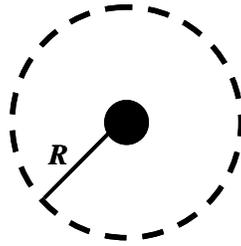


Figure 2. The tree's radius.

We can calculate the circumference of the circle as $2\pi \times 10' = 20' \times 3.14 = 62.8'$ or approximately $62\frac{2}{3}$ feet. Dividing by 12, we determine that each stone is $\frac{2}{3}$ foot long so that

$62\frac{2}{3} \div \frac{2}{3} = \frac{188}{3} \cdot \frac{3}{2} = 94$ stones. This answers the original question, but to construct the circle

accurately, we need the radius R measured from the outside of the tree. Here, $2\pi R = 12'$ so that $R = 1.90'$. Thus, the radius of the tree can be reasonably approximated as 2' so that the working radius R from the outside base of the tree is 8'.

If a Tree Falls...

We encountered a similar problem when cutting down a large oak tree by using an available chain saw which has a 20" blade. The radius of the tree could not exceed 20" so that the tree could be cleanly cut through the center and not left standing precariously. Using a tape measure, we measured the circumference near the base to be 12'.

However, this yields $2\pi R = 12'$. Without a calculator available, we decided to use $\pi \approx 3$, so that $R = \frac{6'}{3} = 2'$.

Unfortunately, this cannot be cut with a 20" blade. Since we know $\pi = 3.14$ is a slightly larger

divisor which will produce a slightly smaller actual radius, we are erring on the side of caution. The solution here was to move the tape measure up the tree until we reached a point where the circumference was 10'. Then, $2\pi R = 10'$ produced a $\frac{5'}{3}$ radius. Converting the fraction to inches produced a 20" radius which provided a safe cutting margin for a perfect cut.

From the Dock, Part I

Oyster (or crab) floats are used to nurture and protect seedling oyster “spats” until they mature, at which time they become marvelous cleansing agents for river and bay water. A quick plumbing problem, related to these floats, involves 4' lengths of capped PVC pipe. How many 2" diameter pipes would it take to replace one 4" diameter pipe for water flow or buoyancy if sealed for air space? Let's consider uniform lengths of 1'. Hence, the 4" pipe has a radius of 2" and a cross section of $2^2\pi = 4\pi$ square inches which yields a volume of 48π cubic inch per foot. For the 2" pipe, the radius is 1". Thus, the cross section area is $1^2\pi = \pi$ square inches, so that the four 2" pipes will produce the same volume per foot. It is interesting to try to draw the four 2" pipes fitting inside of the 4". The four circles inside the 4" diameter circle will overlap in a manner that makes the physical areas difficult to compare. Physically, the four 2" pipes will not fit inside the 4" pipe, thus making physical comparisons non-obvious for a mathematically easy problem.

A similar example involving a comparison of 1" and 2" pipes requires work with fractions, since the radius of the 1" pipe is $\frac{1}{2}$ ". The resulting 4 to 1 ratio of the number of pipes is the same as in the previous comparison.

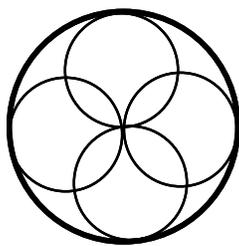


Figure 3. Theoretical PVC pipe placement.

From the Dock, Part II

The aforementioned oyster float part poses an interesting geometrical construction problem. Such floats can be constructed from relatively “stiff,” 1”×1” mesh wire by making careful bends, using a rubber mallet and table edge, to form edges, and then cutting and discarding some corners to allow for closed corners. For starters, think about making an open box by cutting square corners from a rectangular sheet of paper and folding up the edges. The difference here is that the float which will result when the capped PVC tubes are later attached to the sides for flotation must be a closed rectangular solid. We have the capability of clamping edges together with “pig rings” which also can be used to hinge a cut piece of wire on one end to form a “door flap.” One suggested design is similar to the one shown in Figure 4.

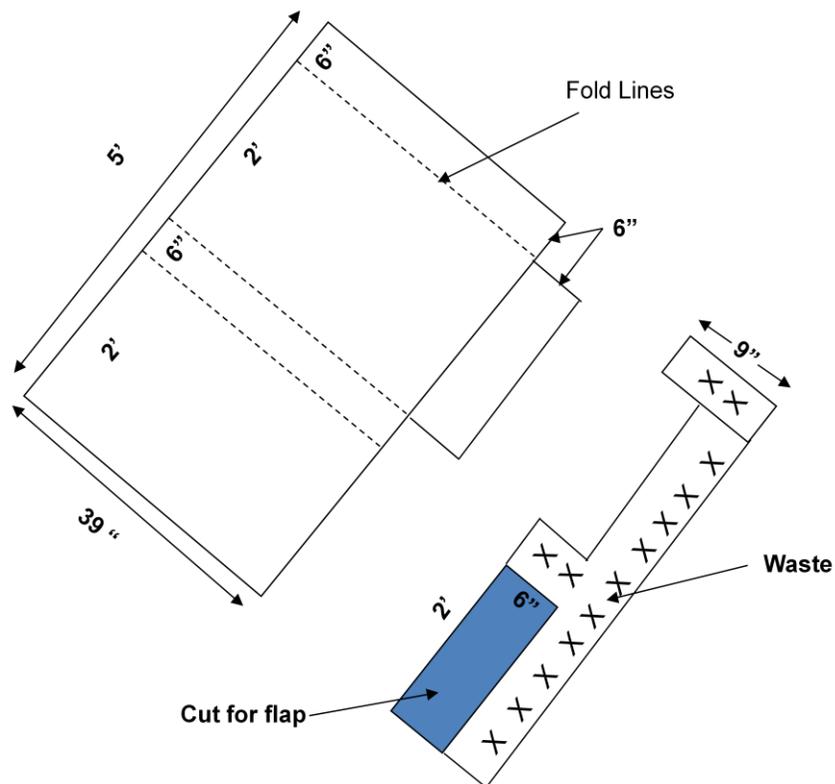


Figure 4. Aerial view of oyster float plan.

When cut, bent, and assembled, what is the inside volume of the float? Calculation in inches yields $24" \times 39" \times 6" = 5615$ cubic inches.

Can you construct a model of similar design using a $5' \times 4'$ piece of 1" coated wire which still has a 6" cage height, but yields a larger enclosed volume? Hint: one solution can be achieved by folding from the 5' side and using a cut from one end to make the flap on the other end. This will produce a volume of 5,832 cubic inches. Although the volume is slightly increased, there might be other dimension considerations which could be pertinent. Can you devise this design or better diagram by folding and cutting a $5" \times 4"$ sheet of paper?

Poolside Problem

A simple volume problem which involves filling a swimming pool with water leads to an interesting gallon-per-cubic-foot equivalence. Consider a swimming pool which has rectangular dimensions of $18' \times 36'$ with an average depth of 6 feet. Multiplying these dimensions in feet yields 3,888 cubic feet, but the problem is that water is delivered in gallons. If you try to guess this relationship, you will probably not be close to the actual answer. An employee of the MathScience Innovation Center in Richmond, Virginia built a reinforced cubic foot model for our geometry class, since no model of a cubic foot could be found on site where metric dominated. The students used a quart container to pour water into the model. They researched the answer and were convinced that 7.48 gallons filled the cubic foot container. Thus, the swimming pool required $3888 \times 7.48 = 29082.24$ gallons of water.

Shingling the Roof

An area problem occurs repeatedly in roofing construction. This problem is usually effective after discussions of the Pythagorean Theorem. How many "squares" of shingles are needed to roof a building with an A-frame roof if the view of the building is like that in Figure 5?

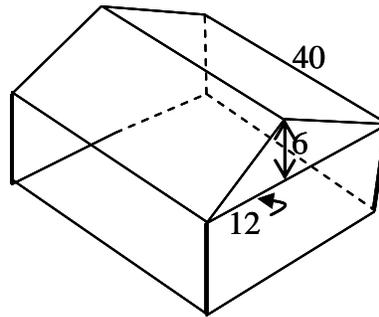


Figure 5. Aerial view of an A-frame roof.

The slope or pitch of this roof is called a 6/12. Using the Pythagorean Theorem rather than making somewhat dangerous measurements from atop the roof, we find the rafter length as $\sqrt{12^2 + 6^2} = \sqrt{144 + 36} = \sqrt{180} = 13.446'$. Since the rafter overhangs the side by 6", we use 14' for the rafter length. Now $14' \times 40' = 560$ square feet which yields 1,120 square feet for both sides of the roof. Since a square of shingles is 100 square feet, we need 11.2 squares. Effectively, we will order $12 \frac{1}{3}$ squares to allow for a 10% cutting loss. This will require thirty-seven bundles of dimensional shingles which require three bundles per square.

House of the Two Gables

Siding the gable ends of a garage with a A-frame roof as in Figure 5 poses an interesting observation which can save half of the possible waste of $4' \times 8'$ sheets of T-111 siding. The sheets of siding have an outside (finished side) which has grooves cut every twelve inches and an inside which is smooth. Thus, the sides are not interchangeable. Obviously, six sheets of siding will cover the $24'$ span on the building front, cutting the tops off on the proper angles. Note that the cuts from the right front will not work on the left front. Can you show, using a piece of paper with lines on the finish outside, how the cut off pieces can be used on the gable on the building back so that we can eliminate waste and need not order six more sheets for the back gable?

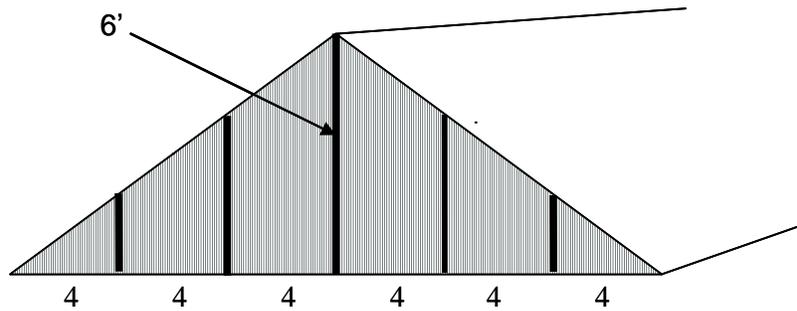


Figure 6. Gable end.

Let It Rain

As a final example, we investigate an actual rain gauge that will measure more than 5” of rain so that the scale in inches is marked approximately $4 \frac{3}{8}$ inches per inch. Since the gauge is nearly 2’ tall, with this large scale per inch, it can be read from a distance of 10-20 yards. The problem is for students to determine the accuracy of the scale by performing actual measurements on the gauge which has a circular collection top much larger than the tube it feeds water into (see Figure 7).

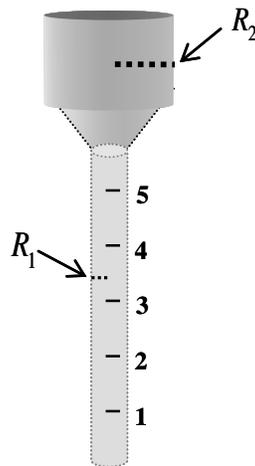


Figure 7. Oversized rain gauge.

Some discussion about how to measure rainfall is useful. Assuming rainfall is uniform in a certain locale, we could use a pot of any dimension, so long as the edges are vertical, to measure

that an inch of rain in the pot represents a rainfall of one inch. Similarly, if the **level** of water in a swimming pool rises one inch, we received an inch of rain, even if the pool is kidney shaped or otherwise. The problem here is to compare the area of the collector top to the area of the collector tube to determine the scale on the tube representing an inch of rain. It is difficult to measure the diameter of the closed tube even when a centimeter tape measure is used. Students usually resort to a somewhat more accurate measurement of the circumference in both places and converting the measurement of the corresponding radii. This provides a good millimeter problem to find each radius since the ratio of the square of the corresponding radii will yield the ratio of the corresponding areas $\left(\frac{\pi R_2^2}{\pi R_1^2}\right)$ and hence, the ratio producing the inch scale on the tube. A

follow-up discovery question is to find a ratio $\frac{\pi R_2^2}{\pi R_1^2}$ which will give an exact inch scale of 4:1. If

the student is familiar with square roots, this will not be difficult. Perhaps a patent could be acquired on a new rain gauge.

Conclusion

While not all of these applications might be interesting to every student, they do illustrate a wide range of mathematical applications. By using these geometrical applications in a practical, hands-on manner, teachers gain confidence in their problem solving abilities which they will, in turn, pass on to their own students. Enthusiasm is infectious, and these students may need little encouragement to find examples of geometry in their own lives. If students are persuaded to present their “findings” during classroom presentations with the promise of extra credit, they may surprise themselves with their own ability to teach.

WHAT COUNTS IN THE PREPARATION PROGRAM OF MATHEMATICS SPECIALISTS AND WHAT LESSONS HAVE WE LEARNED ABOUT WHAT NEEDS TO BE ADDED?

S.S. OVERCASH
Virginia Beach Public Schools
Virginia Beach, VA 23456

Introduction

Five years ago, while I prepared for the transition to my new role as a site-based Mathematics Specialist, I was continually reminded of Tony Robbins' observation: "If you do what you have always done, you'll get what you've always gotten" [1]. During my training to become a Mathematics Specialist, my eyes had been opened to new ways of thinking about teaching math, new ways of learning through collaboration, and new ways of delivering professional development. I also knew that, even before I started this journey, I considered myself a successful teacher and so would many of the teachers with whom I would be working. In order to be effective in improving student achievement through my work with teachers, I understood that it would be important to develop my skills in working with adults, but I knew that would not be enough. I would need to have a vision of what I wanted mathematics instruction to look like and I would need a plan for making that vision a reality. I would need to set goals and be purposeful in planning effectively to meet those goals. This article will describe how the training provided during preparation to become a Mathematics Specialist enabled me to begin my work with teachers in an embedded professional development program. This article will also address what I found to be my own needs for continued professional development in my work as a site-based Mathematics Specialist.

Background

In the *Principles and Standards for School Mathematics*, the NCTM describes "a future in which all students have access to rigorous, high-quality mathematics instruction" [2]. To achieve that vision, teachers need ongoing and effective professional development opportunities. However, the NCTM also admits:

The reality is simple: unless teachers are able to take part in ongoing, sustained professional development, they will be handicapped in providing high-quality mathematics education. The current practice of offering occasional workshops and in-service days does not and will not suffice [2].

As a Mathematics Specialist searching for a guiding vision for working with teachers in site-based, job-embedded staff development the NCTM once again advises:

Imagine that all mathematics teachers continue to learn new mathematics content and keep current on education research. They collaborate on problems of mathematics teaching and regularly visit one another's classrooms to learn from, and critique, colleagues' teaching. In every school and district, mathematics teacher leaders are available, serving as expert mentors to their colleagues, recommending resources, orchestrating interaction among teachers, and advising administrators [2].

Mathematics Specialists provide site-based and in-depth learning experiences for individuals, grade-level groups, and vertical teams which are ongoing, reflective, and close to classroom practice [3]. Schools and school systems have provided professional development for in-service teachers, as well as mentoring programs for new teachers, but these programs have not been reported to be successful in providing for lasting change. The Virginia Mathematics and Science Coalition's "Mathematics Specialist Task Force Report" explains that these programs are not of sufficient duration and often they do not reach all teachers in a school [4]. Instead, administrators and teachers call for ongoing support for teachers "as they move through the continual changes encountered on their journey" of professional growth [4].

The Importance of Data

The definition of a Mathematics Specialist lists one of the seven functions as "interpreting data and designing approaches to improve student achievement and instruction" [5]. One goal would be to use data to guide instruction and also to evaluate at the school level if what we were doing showed a positive effect. While the emphasis on content-focused and data-driven program planning was an important part of the preparation program, I realized that a framework for using data efficiently and effectively had been missing when I went through the Leadership courses. What I discovered was that, even in a school with a dedicated full-time staff member to work with data, I found myself overwhelmed with the amount of data and the amount of time required to make sense of it for planning in a timely manner. When I was invited to participate on the instruction team for a Leadership III class in my fifth year as a Specialist, I was introduced to *The Data Coach's Guide to Improving Learning for All Students* [6]. The procedures and tools included in the guide proved to be excellent resources for identifying student learning problems and organizing data. Additionally, the guidelines for working with data teams have proved beneficial in helping teachers to move past excuse making and blaming to taking responsibility

and collaborating to solve problems. This new resource should prove to be a valuable addition to the preparation program for Mathematics Specialists as they develop skills in interpreting and using data when working with teachers to improve student achievement.

Provide Ongoing Support for Teachers

As a site-based Mathematics Specialist, I wanted to provide ongoing support for all of the teachers in my school. I did not want to be seen as someone who only worked with struggling teachers. I wanted to cultivate the idea that learning is a lifelong process and that professional educators are continually improving their practice. Another goal would be to meet regularly with all teachers in grade-level teams and in across-grade-level groups. During these meetings, we would focus on content, pedagogy, and using data to guide instruction. I would use these meetings as the backbone of my program of professional development to provide a common base of knowledge and experience. My plan included monthly meetings with grade-level teams during common planning times and quarterly across-grade-level team meetings that would meet during time after school created out of a flexible scheduling option. The support of the building administrator proved crucial by adding value to these meetings through her attendance and participation. I felt confident that the training I had received in content and pedagogy during my preparation program would prove beneficial in my work with teachers during these meetings.

Developing a Professional Learning Community

As a site-based Mathematics Specialist, the advantage of working with teachers over time to develop their content and pedagogical knowledge and to provide opportunities for personal reflection on their practice in professional learning communities was a new way of doing things, but one that I had successfully and personally experienced in the preparation program. A goal would be to provide those same types of experiences for the teachers in my school that I had found in the classes for preparation to become a Mathematics Specialist. I wanted to facilitate the development of a professional learning community that would focus on research and best practices while developing content and pedagogical knowledge over time. I realized that my knowledge of establishing and nurturing a professional learning community was limited to being only a participant. In order to facilitate a professional learning community, I used the resources and guidance from the National School Reform Faculty (NSRF): "...professional development initiative that focuses on developing collegial relationships, encouraging reflective practice, and rethinking leadership in restructuring schools—all in support of increased student achievement" [7].

I found further resources and guidance for the use of protocols in *The Power of Protocols* [8]. Additionally, Iverson recommends promoting a Professional Learning Community (PLC) with the following statement:

If schools are to be successful in nurturing professional learning, they must discard the common view of staff development as a series of events in which teachers act as passive recipients of knowledge. The notion of continuous intentional professional learning is based on a constructivist view of teachers' attempts to make sense of their practice by continual exploration of that practice in job-embedded settings [9].

Whether as a resource for personal professional development among Mathematics Specialists or in working with teachers at the school level, explicitly promoting the knowledge and skills necessary to facilitate a professional learning community would add to the preparation program for Mathematics Specialists.

Teacher Beliefs

I soon realized that working with teachers over time required a different set of leadership skills than I had used previously when presenting at conferences or even in one-shot professional development classes at the district level. I started revisiting the *Developing Mathematical Ideas (DMI)* materials used in the content classes for preparation, hoping the facilitators' guides would provide a framework for my work with teachers that would move us toward changing beliefs and the way teachers looked at how students learned [10]. I started wishing that, during my preparation, using case studies and videos as effective tools to lead professional development had been made more explicit. I realized that "mathematics teacher-leaders must themselves engage in ongoing learning and professional development" [2].

Additionally, as a part of the data gathered for the National Science Foundation (NSF) grant through which my position was partially funded, teachers completed a beliefs survey at the beginning of our work together and at the end of each school year during the project. One goal would be to move teacher beliefs along the continuum from a traditional approach to teaching mathematics to a constructivist or reform approach. As reported by Pat Campbell in a presentation at the "What We Have Learned Symposium" in December 2009, the results of the survey data from the schools that participated in the NSF project did show teachers moving away from a traditional belief system about teaching mathematics. However, the data did not show teachers embracing a reform or standards-based belief system. I had facilitated conversations

with teachers and administrators each year after the completion of the beliefs survey. I had embedded the belief statements into grade-level and across-grade-level professional development I created and facilitated with my teachers. I had shared journal articles with my teachers that described learning experiences that utilized best practices described in the belief statements. Even when our participation in the research grant was concluded, the teachers asked that we continue the practice of annually reviewing our beliefs so I knew we had opened up conversations around beliefs. However, I had to admit that I had not been explicit about how our beliefs affect classroom instruction and wondered how I could move teachers in this critical way toward changing their practice as a result of adopting new beliefs. I realized that my leadership skills in changing beliefs were lacking. I knew that if I could not affect teacher beliefs, change in practice risked being superficial and short term. I wondered if there were materials available that would provide a framework for professional development designed to move teachers through a process of constructing a belief system. Iverson describes the scope of this challenge:

One lesson learned is that being a provider of the professional development that will lead to the necessary changes requires new skills and knowledge for the mathematics specialist. Armed with improved content and pedagogical knowledge acquired in university classes, a mathematics specialist will face the challenge of transforming the beliefs, knowledge and habits of practice of both individuals and organizations [9].

I knew that my experiences in the preparation program had changed and clarified my beliefs toward how students learn, but I realized my training had not prepared me for the task of leading others to make similar changes during our work together. I reflected on the experiences that had led to my strong belief in a standards-based approach to teaching mathematics and wondered how I could provide similar experiences for teachers. One idea shared by Pat Campbell at the Symposium would be to use a model of instruction with teachers that incorporates a “stepping in, stepping out” model. The need to be explicit with teachers as a part of modeling, co-teaching, and coaching practice or during professional development opportunities might be helpful in making a connection for teachers between their experiences and their beliefs. I knew this skill was one I needed to explore and acquire as a part of my leadership repertoire.

Strengthening Leadership Skills

As a school-based Specialist, I would need to provide leadership in a variety of ways and would need to develop a variety of leadership skills. Even after nine hours of graduate-level Leadership courses, I realized I still needed to add to my knowledge and skills. As a school-based Specialist, I needed to find opportunities to strengthen my skills working with adult

learners. While the study of West and Staub's *Content-Focused Coaching* provided a successful model for working with individual teachers to improve their practice, what I needed was additional training in providing differentiated coaching for individual teachers [11]. As a Mathematics Specialist, I would need to be able to "demonstrate the ability to identify teachers' individual professional development needs, and individualize staff development efforts to include both formal and job-embedded professional learning experience" [6]. One goal would be to provide differentiated instruction for individual teachers. I found *Differentiated Coaching: A Framework for Helping Teachers Change* to be an excellent resource and would recommend adding it as a topic for discussion during the leadership training [12].

Balancing the Needs of Individual Teachers with School Structure

Trying to balance the needs of individual teachers with the need to make changes to an organization proved to be a daunting task. One goal was to balance my work with individuals with my work with grade-level teams and my work with across-grade-level groups. However, I found myself searching for some guidelines on what a good balance would be. Iverson describes a threshold model for professional development that parallels the work of reading specialists based in schools. From the description of the components of appropriate professional development to the development of individual knowledge leading to student-focused collaboration, this model seems to provide a good framework for Mathematics Specialists and would be a good addition to the preparation program. Additionally, the description of the various dimensions and the idea of a threshold model that moves the organization forward clarified for me how the work of Mathematics Specialists with various groups and in various capacities can work together to meet the common goal of promoting student achievement [9].

Looking Back, Looking Forward

After working as a school-based Mathematics Specialist for five years, revisiting my preparation program was a good opportunity to reflect on what was helpful and what I found I needed on the job. The exposure to programs such as *DMI*, Cognitively Guided Instruction, and *QUASAR* provided models for professional development that I have found useful in my work. The exposure to and participation in a Lesson Study provided one of the most valuable parts of the preparation program. I would consider using a lesson study approach to the work of the instructors in developing content coursework and then incorporating the "stepping in, stepping out" model described above to make the teaching and coaching moves explicit. Exposure to the resources from *The Power of Two* helped clarify the purpose of and model for an effective co-teaching practice [13]. Again, using these materials in a Leadership course and then modeling

and making explicit the characteristics of effective co-teaching would be a valuable addition to the preparation program. The exposure to professional resources in books and journals, along with the sharing of those resources with colleagues, added tremendously to my knowledge base as well as my ability to locate and use resources as needed. With the continued addition of more relevant resources to the literature of coaching, mathematics education, and professional development, I would include even more opportunities for participants to share in study groups, as well as using electronic tools such as blogs or sharepoint sites to share reviews and findings. Additionally, one of the great strengths of the preparation program is the collaboration between various interest groups. At this point in the development of the Mathematics Specialist model, there are Specialists working effectively in schools that can provide insight into the development and refinement of the preparation program and also serve as mentors to Specialists-in-training. While there may not be enough Specialists to serve one-on-one as mentors to pre-service Specialists, perhaps a Teacher-in-Residence model might be a valuable addition to the program. A Specialist working half time in an elementary school and half time as part of the university teaching team could open up new opportunities for improving the preparation program. Fisher's model for a Teacher-in-Residence describes benefits to the district, the university, and also to the preparation program [14].

References

- [1] "Tony Robbins Quotes," Brainy Quotes; Internet:
http://www.brainyquote.com/quotes/authors/t/tony_robbins.html.
- [2] *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 2000.
- [3] *Ensuring Teacher Quality: A Continuum of Teacher Preparation and Development*, WestEd Policy Brief, 2000; Internet: http://web.WestEd.org/online_pubs/po-00-05.pdf.
- [4] "Mathematics Specialists Task Force Report," Virginia Mathematics and Science Coalition Task Force, *The Journal of Mathematics and Science: Collaborative Explorations*, **8** (2005) 5-22.
- [5] "Mathematics Specialists Definition," Mathematics Specialists School and University Partners, *The Journal of Mathematics and Science: Collaborative Explorations*, **8** (2005) 1-2.

- [6] N. Love, *The Data Coach's Guide to Improving Learning for All Students*, Corwin Press, Thousand Oaks, CA, 2008.
- [7] "National School Reform Faculty," Harmony Education Center; Internet: <http://www.nsrfharmony.org/>.
- [8] J.P. McDonald, N. Mohr, A. Dichter, and E.C. McDonald, *The Power of Protocols: An Educator's Guide to Better Practice*, Teacher's College Press, New York, NY, 2007.
- [9] N.R. Iverson, "Mathematics Professional Development that Focuses on Student Achievement: A Parallel Case Example," *The Journal of Mathematics and Science: Collaborative Explorations*, **8** (2005) 119-141.
- [10] D. Schifter, V. Bastable, and S.J. Russell, *Developing Mathematical Ideas*, Dale Seymour Publications, Parsippany, NJ, 1999.
- [11] L. West and F.C. Staub, *Content-Focused Coaching: Transforming Mathematics Lessons*, Heinemann, Portsmouth, NH, 2003.
- [12] J. Kise, *Differentiated Coaching: A Framework for Helping Teachers Change*, Corwin Press, Thousand Oaks, CA, 2006.
- [13] *Power of Two*, University of Kansas; Internet: <http://www.powerof2.org/>.
- [14] W. Fisher, "Teacher-in-Residence Program at California State University, Chico," *The Journal of Mathematics and Science: Collaborative Explorations*, **8** (2005) 143-152.

A WEB OF INFLUENCE: HOW THE MSP PROGRAM HAS SHAPED THE THOUGHTS OF THREE INSTRUCTORS

J. REYES

*Dept. of Mathematics, Norfolk State University
Norfolk, VA 23504*

During the “What We Have Learned Symposium,” the following question was posed to the members of the Content Course Fishbowl Discussion Panel: *How has teaching in this program influenced your own thinking about teaching and learning?*

Given this question, I immediately began to think about the instruction team for the Cohort II course, *Probability and Statistics*. This team consisted of three members: me, Sandra Overcash, and Nancy Wall. In preparing to teach the course, our first task was to revisit the course objectives and subsequently assign roles based on our expertise, interests, and interpersonal skills. It was especially nice to have three faculty members from different backgrounds, therefore not only adding three different perspectives, but also making the delineation of responsibility easier.

As the member with a terminal degree in our content area, it was often my responsibility to give credence to our claim that a deeper understanding of the K-8 content was critical for the participants’ success as a Mathematics Specialist. I also tended to answer the more difficult content-related questions that arose, and strove to keep our delivered content cohesive throughout the summer. In contrast to my status as a relative novice to the Mathematics and Science Partnership (MSP) program, Nancy Wall has taught the class numerous times and has also served as a K-8 instructor for over twenty-five years. Hence, she brought a more intimate knowledge of what goes on inside the classroom, and reinforced our credibility with the K-8 teacher leaders with respect to the Virginia *Standards of Learning* and middle school curriculum. She was also assigned the task of leading most of the hands-on activities and helping us explore common mistakes and concerns with the curriculum. Sandra Overcash was a graduate of the MSP program and, along with Nancy, also served in the master teacher capacity. While she played a lesser role in delivering content, her primary task was to help us make the pedagogical connections within our MSP curriculum whenever possible. She led most, if not all, of the *Developing Mathematical Ideas (DMI)* module discussions; and, based on her own experience in

the program, was able to keep us connected with the needs we were trying to address within our target audience [1].

The main point of the previous paragraph is to point out that, based on our personal experiences, strengths, and weaknesses, each of us brought something quite different, yet equally essential, to the program. Therefore, when answering the original question of what did we *take away* from the program, the question that seemed to naturally follow was, “*How did this ‘take away’ vary from one instructional team member to another?*” It is reasonable to think that our lessons learned and the extent to which we were each changed and influenced by teaching the course would also be shaped by our separate roles and personal experiences. With that in mind, what follows is each member’s account of how the MSP teaching experience influenced their thinking about teaching and learning. I have written each of the following sections from the first-person perspective of the three aforementioned instructors based on a collection of interviews, informal discussions, and e-mails exchanged on the subject.

Sandra’s Perspective: Serving a Community of Learners

As a graduate and subsequent instructional team member for *Probability and Statistics*, it may be fair to say that being an instructor in the program has influenced my effectiveness as a Mathematics Specialist just as much as participating in the program itself. In particular, the three areas in which I was impacted the most were: 1) my level of appreciation for the colleagues whom I coach; 2) my approach to accommodating diverse learning styles and mixed levels of preparation; and, 3) a better understanding of the value of cooperation.

Teaching the class has played a large part in shaping how I handle my responsibilities as a Mathematics Specialist. In thinking about the course *Probability and Statistics*, I clearly remember the extent to which we continually challenged the participants to grow in their ability to think deeply and challenged their preconceptions of how math should be taught. For some participants, this was a struggle; and, it was impressive to see them rise to the occasion as we seemingly made a habit of redefining what they thought of as “teaching best practices.” With that in mind, when working with my own teachers during the school year, I have often thought about how much I am asking of them. Unlike the MSP participants, these are not teachers who are getting credit for a class or have an expectation of a new job title as a consequence of our interaction. Consequently, when I reflect on how difficult it was at times for us as the instructional team to stretch the minds of teachers who were *willing participants* in the learning experience, it serves as a constant reminder of the amount of patience I must continue to exhibit

as I serve in the capacity of a Mathematics Specialist. Such reflections also help to reaffirm the amount of gratitude I have for each teacher's willingness to work with me to build a better product for our students.

As for more tangible ways that the program has influenced my teaching and learning, I also remember how difficult it was at times to design and deliver a lesson that would meet the needs of each participant in the room *simultaneously*. From the kindergarten teacher to the eighth grade teacher, veteran to novice, K-2 certified to middle school certified, tactile to audio to visual learners; the backgrounds of our participants varied tremendously. While we expect such variation in regards to the students in our K-8 classrooms, I suspect that this may be overlooked by Mathematics Specialists when working with our colleagues. However, seeing how our instructional team rose to this particular challenge has given me better perspective on the needs of my teachers, and has shaped how I fulfill my role as a coach.

As an example, one activity that I fondly remember from *Probability and Statistics* is the lesson on tree diagrams. Jerome did such an outstanding job of taking what I thought would be a rather simple lesson and making it an activity that challenged everyone and caused them to question their understanding of the concept, as well as the strategies used to teach it. I primarily work with teachers on a one-on-one basis when coaching, but seeing the dynamics of the participant group and how they interact with each other reminded me of how diverse a given group of teachers can be in terms of their receptiveness, learning styles, and their ability to absorb new concepts. It also reminded me of how a well-crafted lesson can work to meet each (or at least most) of them where they are on the learning spectrum. As I am working in classrooms, I often keep the tree diagram activity in mind and use it as a reference point for how to construct my lessons. It serves as a reminder to always equip myself with ways to make an activity more challenging if needed, as well as being prepared to alternate ways to present the concept, whether for the entire class or individuals.

Along those lines, I am also reminded that in planning to teach *Probability and Statistics*, the task of figuring out how to sequence topics and activities in a manner that best suited short and long term retention, and being able to steer a group of adult learners without completely taking over was hard work! In watching the participants interact with each other and with the instructors, and knowing first-hand the amount of thought that went into each lesson's development, I am reminded of how critical it was to be respectful of the experiences of the participants while helping them to be reflective on their own knowledge and skills. This is a skill

that is tricky at best, but certainly one that I hold at the forefront of my mind as I continue to shape my own thoughts about teaching and learning.

Finally, the third reflection I have is on the co-teaching model. In our *Probability and Statistics* class and also the leadership class that I co-taught, there was a large investment of time and energy to plan for each class. Even though the class is pre-designed and has been taught previously, with every new cohort there is a need to think about the strengths and weaknesses of the specific group of participants *and instructors* for a particular course—and to revise and refine what we are doing accordingly. In the end, the stellar results made it clear that the co-teaching model is a valuable tool that is worth the investment, as it combines the strengths and unique perspectives of multiple instructors in order to deliver a product unique and custom tailored to the given audience. Despite previous experiences to the contrary, it became clear to me that the co-teaching model should go well beyond a simple delineation of duties. As a result, I have since sought to incorporate a similar strategy when coaching that will likewise draw on the advantages of a cooperative team.

Although I am a full-time Specialist, I do still teach classes of students on occasion. Fortunately, someone is always willing to share their classroom of students with me as I look to stay connected with the students while also figuring out the extent to which certain strategies and new ideas will work. Having such a worthwhile experience with the co-teaching model in the program has left me little choice but to ensure such co-teaching opportunities are well thought-out, well synchronized, and customized to the given subject, teacher, and audience. In doing so, my goal is to model for the students (and perhaps their teacher as well) the extent to which tacking a common agenda in a properly planned partnership can pay incredible dividends. This idea of enhanced synergy, together with the aforementioned notions of planning for diverse learners and furthering appreciation for teacher students, are clear evidence of the large extent to which the MSP program has impacted my approach to teaching and learning. For that, I am tremendously thankful.

Nancy's Perspective: Examples That Dig Deep

The textbooks are broken, and the exams don't help. In ten words or less, that is how I would summarize what I've learned from teaching in the MSP program. In the paragraphs that follow, I'll explain how I came to this opinion, as well as how it has changed my approach to teaching.

Although I've been in the classroom for over twenty-five years, it admittedly doesn't take long to realize that most students are susceptible to the same trap doors in mathematics. That is, after teaching any particular lesson multiple times, the errors that the students will make and the specific points that will consistently lead to confusion quickly become evident to the instructor. For instance, when middle school students learn about absolute value, many in the classroom will simply associate this operation with the notion of "reversing the sign." With that, I can usually guarantee that a handful of students will tell me that $|+3|$ must be -3 . I typically presume that this tendency to jump to erroneous conclusions is a byproduct of a student's desire to find the "shortcut." We can often associate this mindset with students who ask questions such as, "will it always be this way?" Or, "I got the same answer by doing this, this, and that. Is that going to work every time?" In these cases, my guess was that these students were simply too anxious to establish a pattern in hopes of eliminating the need to understand on a deeper level. I assumed that such errors were solely a product of the student's approach to learning. However, teaching in the MSP program has shown me otherwise.

After working with MSP participants over multiple years and programs, I was surprised to find that the aforementioned pitfalls and traps for confusion (albeit different and at a higher level of content) were almost as predictable with the participants as with my middle school students. The difference, however, was that the participants were able to better communicate what they felt was the source of their confusion. Almost every time, it could be traced back to either the manner in which a particular concept was presented in their textbooks or the manner in which it was to be tested on a standardized exam.

In an effort to maintain the "Three C's of Instruction" (lessons that are clear, concise, and consistent) it seems that many textbooks and exams recycle the same examples to illustrate the concepts within. Unfortunately, while these Three C's may indeed be achieved, using the same textbooks, same examples, and same exam structures over multiple years has arguably led to a very narrow scope of understanding for students and teachers alike. Students and participants consequently seem to have trouble transferring examples to scenarios beyond what has been presented. This has only been exacerbated by standardized tests such as the Virginia *SOL*. With these tests in mind, we as teachers have often limited the scope of a topic even further, tightening the curriculum to the point where neither students nor teachers think outside of the proverbial box. Therefore, connections between topics, the ability to extrapolate concepts, and any sense of "a big picture" are lost. It has also led to a more microcosmic view of the material. For example,

we have suddenly become more concerned with how to label a graph rather than how to interpret it; perhaps, simply because the former is easier to assess on an exam.

Through countless conversations, “ah-ha moments,” and the sharing of similar classroom experiences, the participants in the MSP program showed me that it is these confines that have created many of the traps and pitfalls mentioned earlier, and not necessarily a hurried approach to learning. To further illustrate the point, suppose that we were to put five teachers in a room, and ask each of them to provide three examples to be used to teach the conversion of pie charts to frequencies. My guess is that we may be surprised at how similar all five problem sets would be with respect to context, phrasing, and even the “nice numbers” used as denominators, etc. I base this guess on the fact that, for many of the topics that we covered, the points of confusion for the MSP participants seemed to be consistent across the class. The problem is that, as a group, by using the same tried and true examples to teach and learn the mathematics, we have set up confines for our thought processes without even realizing it. These confines have often limited how and when we think a given concept can be applied. This undoubtedly leads to the aforementioned false patterns, erroneous shortcuts, and misconceptions of the material presented.

The rich conversations that transpired in the MSP classrooms have caused me to ask more questions concerning the context *surrounding* the concepts that I teach. I now see how critical this can be when it comes to debunking erroneous associations and misconceptions within the mathematics. For example, in teaching graph types some important questions to ask are, do we (via our lessons, texts, or exams) limit ourselves to categorical data, or does the set of accessible examples extend to numerical data? Were we sure to show a scatter plot that *doesn't* show a trend, or did we only focus on the fact that trend lines can show positive or negative correlation? Did we discuss the calculation of median if there are more than 20 data points (because according to many books all experiments apparently stop at 10 or 20 observations)? For line graphs, do we always use years on the horizontal axis, or have we also included other measures of time? Again, while such questions may seem unsophisticated, it was eye opening for me to hear questions such as, “Don't the time units have to be seconds? All of the examples I've worked with show it in seconds, so I figured it had to be.” While these comments may be troubling, it is a prime example of what we should expect when the scope of a given subject has been consistently funneled down to its simplest form.

In all, these considerations have reshaped how I teach my middle school students, as well as how I approach learning for myself. As I reflect on my MSP teaching experience, I am now

sure to take the students beyond the examples in the book. Rather than the blind application of exercises presented, I look for and often design examples specifically to debunk as many of the erroneous shortcuts as I can. I do everything in my power to ensure that the presented skill set is transferrable beyond the confines of the standard three examples used for a given topic. I look for patterns in the work presented and ask myself, when does this work, and when does it not? Being a part of the program and having the opportunity to work and share experiences with such wonderful teachers has reminded me that, whenever possible, I have to teach and likewise strive to learn for understanding beyond the scope of typical yet often mundane examples and exercises. If these perils of learning can affect the MSP program participants (who are some of the most talented teachers in the world), then they can certainly create havoc in the learning experience of a middle school student.

Jerome's Perspective: A Hands-on Experience

If there was one word that was added to my vocabulary as a result of working with the MSP program, it would be “manipulatives.” While anyone with the slightest bit of experience in the field of Mathematics Education knows this term well, as a college professor with an engineering degree and no K-12 teaching experience, it was not a word that I heard outside of the occasional conference presentation on pedagogy. Simply put, most of us teach as we were taught, and not as we were taught to teach (if we were taught to teach at all); and, manipulatives were simply not a part of my personal learning experiences.

With only my traditional style of teaching to fall back on, I was more than happy to simply push content forward in the lecture format that most would expect in a college course, allowing for brief discussions when deemed appropriate, and the occasional break to work examples as a class. However, those sentiments began to change just a few days into teaching in the MSP program. In *Probability and Statistics*, every lesson was centered on an activity that involved a manipulative of some sort. While my buy-in was not immediate, I was fully sold by the end of the first course. The importance of hands-on activities as way to express and understand abstractions, as well as a tool to facilitate cooperative learning was suddenly obvious. In a presentation on virtual versus hands-on manipulatives, Hunt, Nash, and Nipper point out the advantages of manipulatives [2].

- The relationships formed (between concepts) by the use of manipulatives incorporate visual, tactile, and kinesthetic experiences.

- The use of manipulatives and activities add a component of “cooperative learning and reflective discussion [that] further enhances depth of understanding and the likelihood of retention [3].
- The authors quoted Moyer, et al. as stating, “Because it is advantageous for students to internalize their own representations of mathematics concepts, interacting with a dynamic tool during mathematics experiences may be much more powerful for internalizing those abstractions” [4].
- Promotes (learning and) teaching by more than just modeling a procedure on the chalkboard [5].

As the MSP program participants wrestled with new concepts and strove to mentally organize the content, I was fortunate enough to see all of the aforementioned components at work. With that, I not only embraced the concept of manipulatives and activity-based lessons, but quite possibly went overboard in my attempt to implement these aspects into my university curriculum. I returned to my classroom in the fall waving the flag of manipulatives, and seeking to implement hands-on activities into every lesson. It was not long before I hit a roadblock. The problem was that I was trying to add time-consuming activities to a pre-designed shared curriculum that *was not built* around such practices. Arguably, it was built to squeeze more than twenty weeks of material into a 15-week semester. In this system, pace was critical and lecture time was a priceless commodity. As a result, in trying to transform a curriculum overnight, I struggled with time management and completion of course objectives. It did not take long to realize that I had to come up with a different means to produce the same end. With little room to add to the curriculum, the ideal strategy would have to utilize components that were already embedded in the curriculum in order to achieve the desired outcome.

This quest for curriculum reform led to what I lightheartedly called the “rebirth of the high-numbered exercises.” In most textbooks, these are the problems at the end of the section that are typically in paragraph form and involve “applied examples from fields such as business, pop culture, sports, life sciences and environmental studies that show the relevance of [the given topic] to daily life” [6]. Simply put, these are the exercises that we usually call “word problems.” Through discussions with colleagues and students, and after thinking about my own experiences, I realized that student disdain (and subsequent faculty frustration) for word problems has slowly phased out many teachers’ willingness to tackle these more challenging problems at the end of the section. Yet these are in fact the exercises that (when well chosen and well presented) serve to: 1) bring the curriculum to life; 2) make the content relevant to topics outside of the

classroom; and, 3) promote discussion of the mathematics. These three attributes can serve to promote increased motivation for learning, improve long-term retention of content, and foster a cooperative learning environment. Thus, the hope was that an increased emphasis on these exercises would do well to mirror the previously mentioned advantages of manipulatives that I experienced, thus creating a replica of the MSP learning environment of which I was so fond.

However, to achieve these results, it was not enough just to reinforce the value of word problems. The order of the curriculum also had to be shuffled. I addressed this point at a recent Virginia algebra conference during a presentation entitled, “A Dramatic Approach to Real World Connections.” The theme of our discussion was, “beginning with the end in mind” and during the presentation, we highlighted the use of role play and scenario building to motivate student learning and improve ability to connect topics within and across disciplines. The idea was to stop using the aforementioned word problems solely for assessment purposes. Instead, we proposed that these problems should be presented and discussed *before the lesson was taught*. In this manner, they could be used as a tool to achieve all three of the objectives enumerated in the previous paragraph. By presenting the students with a problem that on the surface seemed worth exploring, yet for which they lacked the tools to solve, interest in the upcoming section(s) increased tenfold, discussions abounded, and my classroom suddenly had the buzz of an MSP course. In addition, any negative effect on curriculum pacing was minimal as these were exercises that were already slotted to be covered (though often skipped) in the curriculum. However, given different placement and more emphasis, these exercises were transformed from being perceived as an unnecessary hurdle to jump at the end of each section to being the pinnacle of class discussions and motivation to continue learning.

Even beyond this particular strategy, what I have found was that teaching in the MSP program has invigorated my willingness to consistently evaluate my approach to teaching and strive for better results. As I sit in the back of my precalculus classroom, listening to my students engaged in a passionate discussion about Mount Everest, their unwillingness to climb it, and the need nonetheless to know how tall it is (a topic that is squarely based on an exercise in their textbook and will lead us directly into a lesson on the usefulness and application of inverse trigonometric functions) I listen intently and smile, appreciating that the MSP program has reminded me that teaching and learning can be so much fun.

A Web of Influence

For weeks, Nancy, Sandy and I worked together diligently to devise a course that would best serve the needs of the MSP program participants. Yet only in hindsight did we have any clue of the extent to which the relationship between instructors and participants would be symbiotic. Collectively, our three perspectives remind us of the extent to which the benefits of an MSP program are multifaceted. Though we usually focus on the success of our primary product (highly qualified Mathematics Specialists that will positively shape the lives of many) our personal experiences are a reminder that the depth and richness of the program continues to multiply even beyond our program participants as we (the instructors) also take these lessons learned back to our respective workplaces.

References

- [1] D. Schifter, V. Bastable, and S.J. Russell, *Developing Mathematical Ideas*, Dale Seymour Publications, Parsippany, NJ, 1999.
- [2] A.W. Hunt, L.E. Nash, and K.L. Nipper, *Virtual versus Hands-on Manipulatives in Teacher Education: Is One Type More Effective than the Other?* Presentation at Association of Mathematics Teacher Educators, Orlando, FL, 2009.
- [3] H. Daniels, A. Hyde, and S. Zemelman, *Best Practice: New Standards for Teaching and Learning in America's Schools*, Heinemann, Portsmouth, NH, 1993.
- [4] P.S. Moyer, J.J. Bolyard, and M.A. Spikell, "What Are Virtual Manipulatives?" *Teaching Children Mathematics*, **8**(6) (2002).
- [5] B.M. Vinson, J. Haynes, J. Brasher, T. Sloan, and R. Gresham, "A Comparison of Pre-Service Teachers' Mathematics Anxiety before and after a Methods Class Emphasizing Manipulatives," Paper presented at the Annual Meeting of the MidSouth Educational Research Association, Nashville, TN, 1997.
- [6] L. Lial, J. Hornsby, and D.I. Schneider, *College Algebra and Trigonometry*, Addison Wesley, 2008.

COACHING: ONE MATHEMATICS SPECIALIST'S STORY

C.B. DOYLE
Richmond Public Schools
Richmond, VA 23219

It's Monday morning. I walk into the office, speak to the office staff, and get my key. I turn to open the door and hear, "Ms. Doyle, Ms. Crane's biweekly data doesn't look good. I want you to work with her." This is a very familiar request that I have heard numerous times from my principal. So, I place a note in Ms. Crane's mailbox letting her know that I would like to meet during her planning period the next day.

I hadn't worked with Ms. Crane on an individual basis and was hopeful that she would see our working together in a positive light. What I knew of her was based on what I had observed during quick walk-throughs in her classroom and during grade-level meetings. I knew that she had been teaching for four years and had taught only fifth graders. Her classroom was set up in rows, and from what I had observed, she was a very traditional teacher. She used manipulatives occasionally, but even these were used in a very traditional, scripted manner with little student exploration.

As I walked up the three flights of stairs to my office, the word "relationship" kept coming to mind. In my Education/Leadership courses, the relationship between the coach and the teacher was an important topic. My instincts told me that I might need to devote a little extra time to the relationship aspect as I approached this new coaching endeavor. Since I was assigned to work with Ms. Crane, I needed to make sure she realized that the focus of our working together would be to assist her in modifying and adjusting her instruction so that all her students are able to learn the mathematics content. It was very important that she saw me, not as an evaluator, but as a "partner" working to provide the best instruction so that all of the students can become mathematically proficient.

I looked on my book shelf and found my copy of *Content-Focused Coaching* by West and Staub [1]. It would be great if we could do all of the components: the pre-conference, observation, and post-conference. However, I knew from experience that two out of the three would be an accomplishment. I had previously highlighted different sections of the book, and

located the section entitled, “Getting to Know the Teacher.” I wanted to have specific questions in mind for our first meeting. As my eyes roamed over the page, I saw that the first highlighted sentence read: “The purpose of the first conversation is to establish a mutual agreement to work together, to begin to define the parameters of that work and to lay out a plan of action, or at least a framework that feels comfortable and productive to both parties” [1].

“Wow,” I thought, “that’s a lot for one possibly twenty-minute conversation.” I decided to begin with only four of the suggested questions or topics. I selected the following because I thought they would lead to an informative discussion:

- What is your favorite subject to teach?
- What are your feelings about math?
- What is your math history?
- Tell me about your students.

Tuesday afternoon arrived and I headed to Ms. Crane’s room. We exchanged pleasantries. I had a flashback to the *Content-Focused Coaching* book. I had highlighted “...that there be no hidden agenda.” Should I tell Ms. Crane that I was told to work with her because her biweekly data was low? Would it be dishonest not to mention this? As if she read my mind, she said, “I know my biweekly data wasn’t the best.” I let her know that the principal had shared this with me. I asked her to tell me why she believed her students scored so poorly. She explained that she had been out three teaching days and hadn’t taught all of the skills that were tested. However, her students had caught up and she had retested them. She shared the students’ tests; they had done very well on the multiple-choice assessment. I indicated that I still wanted to work with her on the next strand even though she might not need assistance with teaching the fraction concepts. She responded with a slightly reluctant, “That will be fine.” I ignored her hesitation and began to ask some of the questions that I had selected. I learned that reading was her favorite subject. She indicated that mathematics was “okay” and that “sometimes it was fun to teach it.” She had a minor in history and described herself as “having very little math” in school. Ms. Crane considered her students to be very good readers. She said they enjoyed doing math problems on the board and that they were very active. She also mentioned that they could get very noisy and boisterous when she tried to put them in groups or partners, and this was her reason for using row seating. Ms. Crane said that all of her students do well except for three or four that “don’t seem to get it” unless she pulls them aside during her planning period.

I confirmed that geometry was a future strand that she was to teach. She said that she intended to use her geometry plans from the previous year. I suggested that she bring them to our next meeting so that we could adapt them to meet the needs of the three or four students that usually “don’t get it.” When I asked her for the best time to meet before her first geometry lesson, I was surprised when she suggested the next day during her planning period.

During the course of the day, I reviewed the NCTM *Principles and Standards* for geometry, the Virginia *Standards of Learning* objectives, as well as my school system’s Curriculum Compass [2, 3]. This was a strategy that I learned from the Leadership courses. Each time we took a content course, we followed up in the Leadership course by investigating the NCTM *Principles and Standards* and the *Standards of Learning* objectives for the strand. I continue to do this because it provides a clearer picture of the prerequisite knowledge that the students need and gives me a better understanding of what the true conceptual knowledge should look like.

I also referred back to *Content-Focused Coaching*; specifically, the core issues in designing a mathematical lesson. I thought that these would be good questions to keep the conversation focused on the mathematics content and student learning. I decided to focus my questions on: 1) the goals of the lesson; 2) students’ prior knowledge and difficulties; and, 3) how the lesson helps students reach the goal. There were so many interesting questions to choose from that would provide a great deal of insight into Ms. Crane’s teaching and beliefs about student learning, but I thought that if I tried to focus on too many I wouldn’t be able to devote enough time to each one. Time is always a factor when working with teachers—there never seems to be enough of it!

The Pre-Conference

On Wednesday, Ms. Crane brought one lesson plan to our pre-conference. It consisted of pages from the teacher’s edition of a mathematics textbook. She indicated that this was the lesson she had planned to teach. It was a lesson plan on congruence. There was a warm-up activity, teacher modeling, a practice page, an independent workbook page, and a closing activity. As I glanced over the lesson, I remembered the textbook lesson that I had adapted during the Leadership course. It was very similar to this one. I am still impressed by how much of the work I did in the Leadership course continues to be relevant in my day-to-day work as a Mathematics Specialist.

Looking over the lesson, I came to realize that we needed to do a great deal of work with it to make it more effective. I also realized that I would have to provide some effective questioning in order to guide Ms. Crane so that she could understand how the lesson could be changed to meet the needs of *all* of her students. In addition, I had to make sure that the beginning of our working relationship remained positive. I didn't want Ms. Crane to feel that I didn't see any value in her selection of a lesson plan.

Before we started discussing the lesson, Ms. Crane let me know that she only had twenty minutes of planning time. The time factor appeared yet again. This didn't give me very much time for effective questioning or discussion. I decided to concentrate on the goal of the lesson and the students' prior knowledge and possible difficulties. I believed that discussing these issues first would be a good way for her to begin incorporating her students' specific needs as she planned her lesson. Even though we might not adapt the actual lesson, I would feel that we made some progress if I could get her to think differently about her approach to the lesson.

Ms. Crane described the goal of the lesson was for students to identify congruent, non-congruent, and similar figures. She indicated that her students could identify shapes that were the "same," but didn't understand the terms congruent, non-congruent, or similar. When I asked her about areas that her students might have difficulty with, she indicated that the vocabulary might be a problem for about four of the students and she listed them by name. I asked her to show me where in the lesson plan she would provide the necessary assistance for these students. She glanced through the lesson, and then responded that she would have to add it in, "probably while the other students were doing the textbook page." I knew that our time was up, so I asked her to think about what type of assistance she would give and if she would use manipulatives to supplement the textbook with this group. I hoped that these questions would provoke some thought about using manipulatives rather than just the textbook.

We really could have used at least an hour and a half or more to discuss the lesson, but I felt that I had given Ms. Crane some important issues to think about in this short amount of time. I let her know that I was looking forward to observing her lesson on Thursday.

The Lesson

I was excited about observing Ms. Crane's lesson. I wondered what type of support she would provide to those students she identified as needing additional assistance. Would she attempt to make any changes at all?

Ms. Crane began the lesson as outlined in the teacher's edition. The lesson was very traditional. The students repeated and recited the definitions for the vocabulary. Ms. Crane showed the students how to check the figures visually for the same size and shape and then refer back to the definition to decide if the shapes were congruent, non-congruent, or similar. As I observed the lesson, I wished that there had been more time in our conference to discuss using tracing paper and questions that she could ask to lead the students to form their own definitions. Even though the lesson didn't demonstrate best practices, the majority of the students showed an understanding of the concepts.

Situations like this usually pose a dilemma for me. It's true that I wanted the students to gain as much understanding from the lesson as they possibly could—but how will a teacher begin to realize that her instruction isn't building mathematical thinking and conceptual understanding in her students, especially when the students are able to demonstrate mastery on assessments? In many cases, I have found that most teachers determine the success of a lesson based on test scores and believe that, if the students are passing, there is no need for changes in instructional delivery. As a Mathematics Specialist, I often wonder how I can get teachers to understand that it is possible and valuable to teach for conceptual understanding and still have the students do well on assessments.

As I continued to observe the lesson, I noticed that Ms. Crane had been fairly accurate in identifying which students would have difficulty. She pulled those students to a round table away from the whole group. I expected her to go over the textbook page. However, she took out vocabulary picture word cards and used these to review the concepts. The vocabulary picture word cards had the definition and an example. She also had cut out shapes in different sizes for the students to categorize and describe as congruent, non-congruent, or similar. As I watched, I thought that this would have been a great activity for the whole class if they could have also worked in small groups. As Ms. Crane continued with the small group lesson, my mind jumped to envision the students working in pairs and discussing how to compare the shapes. However, the small group lesson proved to be very traditional as well. Ms. Crane talked the group through comparing the shapes just as she did with the whole group. One student, however, took one shape, placed it on top of another, and then observed that it was congruent. "Yes, great strategy!" I thought. I had hoped that Ms. Crane would use this as a teachable moment and let the students discuss this strategy and possibly share others; however, she proceeded with the lesson as before.

After Ms. Crane completed this activity, she sent the students back to their seats to complete the textbook page.

Many questions came to mind as I observed the small group interaction. Was she unsure how to use manipulatives to bring out conceptual understanding? Did she not encourage the new strategy because she didn't know what questions to ask? How would she have responded if I had chosen to interject with a question?

The Post-Conference

Ms. Crane sought me out during her lunch break. She was anxious to know what I thought about her lesson. I didn't want to put her off, but I needed more than ten minutes for us to discuss my observations. I commented on her selection of manipulatives and asked if we could meet after work. She agreed.

I decided to focus only on the small group portion of the lesson unless the other areas came up during the course of the conference. I began the conversation by asking Ms. Crane how she thought the lesson went. She thought the lesson was effective. She shared that she believed that the students understood the concepts, especially the small group. I asked her how the students showed her that they understood. She explained that she watched them categorize and group the shapes, and they completed the textbook page correctly. She showed me the textbook work where the students wrote the words congruent, non-congruent, or similar. I then asked her to explain what in her lesson helped those particular students she thought were going to have difficulties mastering the concepts. She indicated that it was pulling them aside and using the cutouts and vocabulary cards. I asked Ms. Crane what her thoughts were about the student that had put one shape on top of another. She indicated that she saw it, but that pursuing it would get the students off task. She indicated that the students couldn't do that with the *SOL* Test. I used this opportunity to suggest that the students could trace the shapes as another strategy. I offered to come to her class and model a lesson on congruence using different strategies. She asked if I could come in the next day since that would be the last time she would be teaching this concept. I usually prefer more time to prepare, but I felt that it was important not to let this opportunity pass, so I agreed. Ms. Crane began to look at her watch and stated that she really had to leave. I felt that we had just gotten started, but it was after school and we were on her personal time. As she was leaving, I posed a question: Would the small group activity have been as effective if she'd used it with the whole class? I also asked her to think about how the small group lesson may have been different had the students worked in pairs to discuss and categorize the shapes.

Conclusion

This initial coaching experience with Ms. Crane opened the door to more coaching opportunities with her. Even though I would have preferred to have had more time for lesson co-planning and conferencing, the work that we did set the foundation for our extended coaching relationship. I realize that there was so much more that needed to be addressed; but realistically, a Mathematics Specialist has to take the time she is given and prioritize which topics to address. It would have been wonderful to get more involved with the mathematics and adapt the lesson, strategies, and tasks that were used. However, even with the time constraints, I believe that the questions that I posed provided the opportunity for Ms. Crane at least to start thinking about student learning as she plans her lessons. Since this first coaching opportunity, we have worked together many times and she has responded in a positive manner to my suggestions. We have looked at student work, co-taught together, and I have modeled lessons for her. I realize that if I had tried to cover all of her weak areas at one time, she might not be as receptive as she is now to further coaching. Ms. Crane continues to be open to trying new strategies and looking at her students' learning differently.

References

- [1] L. West and F.C. Staub, *Content-Focused Coaching: Transforming Mathematics Lessons*, Heinemann, Portsmouth, NH, 2003.
- [2] *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 2000.
- [3] *Standards of Learning for Virginia Public Schools*, Board of Education, Commonwealth of Virginia, Richmond, VA, 1995.

PROVIDING REAL-WORLD EXPERIENCES: THE VIRGINIA TECH EXTERNSHIP FOR MATHEMATICS SPECIALISTS

B. KREYE and J.L.M. WILKINS
*School of Education, Virginia Polytechnic Institute & State University
Blacksburg, VA 24061*

Abstract

We describe the structure and implementation of the yearlong Externship experience associated with the Mathematics Specialist program at Virginia Polytechnic Institute & State University (Virginia Tech). We discuss the assignments and experiences included in the Externship, the alignment of those experiences with the job description developed by the Virginia Mathematics and Science Coalition Task Force, and teacher comments on the effectiveness of their Externship experiences [1].

Introduction

The Mathematics Specialist program at Virginia Polytechnic Institute & State University (Virginia Tech) is a three-year master's degree program designed to provide a cohort of practicing teachers with the content, curriculum, leadership, and assessment courses required for certification and licensure as a K-8 Mathematics Specialist in Virginia. During the third year of this program, teachers are engaged in a yearlong externship experience. This Externship was designed as a capstone experience for teachers to provide practical experience associated with being a school-based Mathematics Specialist within the teacher's home school setting. The recommendations and final report from the Virginia Mathematics and Science Coalition Task Force served as the framework during the design process for this Externship [1]. The specific requirements for the Externship during the fall and spring semesters are presented in Table 1. Each of the specific requirements within the Externship was chosen to align closely with the roles and responsibilities set forth in the Task Force recommendations [1].

Table 1
Requirements of Externship

REQUIREMENT	DESCRIPTION
FALL SEMESTER	
needs assessment	This report included an outline of the mathematics teaching and learning needs of the school and how they relate to the project to be conducted for the Spring, a review of literature related to the chosen project, project implementation plan, and expected outcomes (personal and school-based)
principal meetings	As part of the needs assessment, teachers were to meet with their principals during the yearlong Externship to discuss needs and issues.
grade-level meetings	As part of the needs assessment, teachers were required to meet with each grade level to discuss hopes and needs for math.
lead a faculty or department meeting related to math	As part of the needs assessment, teachers were to take on a leadership role in a faculty or department meeting based on discussions with the principal.
contribute to the implementation of a district-wide initiative	The cohort was required to develop a professional development experience for all teachers in the district related to the <i>Enhanced Scope and Sequence Plus</i> document.
thought papers/ reflection	Throughout the semester classes, teachers responded to writing prompts designed to have them reflect on different aspects of the needs assessment process and experiences.
activity log/time log	Teachers were required to keep a log listing the types of activities engaged in as a Mathematics Specialist and the approximate time associated with each activity.
SPRING SEMESTER	
final project	Based on the needs assessment, each teacher was required to conduct a final school-based project to address the determined need. This initiative was to represent a systemic program—be more than a “one-shot” initiative.
report of final project	This report included the background literature review for the chosen project, rationale for the project, needs assessment, implementation plan, anticipated outcomes or research questions,

	methodologies/procedures, findings/outcomes, an overall discussion of the project, its impact and implications for future plans.
presentation of final project	Teachers were to present a 20-30 minute presentation of the chosen project—background literature, needs assessment, rationale for project, plan implementation, results, evaluation of the project, and implications for the findings of the project.
activity log/time log	Teachers were required to keep a log listing the types of activities engaged in as a Mathematics Specialist and the approximate time associated with each activity.
thought papers/reflections	Throughout the semester, teachers responded to writing prompts designed to have them reflect on different aspects of the needs assessment process and experiences.

Experiences Prior to and Concurrent with the Externship

The seventeen teachers that were involved in the initial Externship experience were in their third and final year of the Mathematics Specialist program at Virginia Tech. Fourteen of the participants were teachers from a local school district and three were acting Mathematics Specialists within a city school district. Prior to the Externship, the teachers had completed five mathematics content courses, as well as two courses entitled *Assessment in Mathematics* and *Mathematics Education Leadership I*. Through the leadership course, the teachers learned about content area coaching and mentoring, and each was involved in the full coaching process (interview, pre-conference sessions, lessons, and post-conference sessions) with both a new teacher and a veteran teacher at each of their schools [2]. At the beginning of *Leadership I*, the principal of each teacher was visited by university faculty to discuss and describe the structure and expectations for that course.

Concurrent with the yearlong Externship, the teachers were enrolled in the fall in a course entitled *Advanced Curriculum and Instruction in Elementary and Middle School Mathematics*, in which they were involved in conducting a formal lesson study with a group of their peers in their home schools. As part of this course, teachers were also required to present a session (individually or in a small group) at the local annual conference of the Blue Ridge Council of Teachers of Mathematics. In the spring, they were concurrently enrolled in *Mathematics Education Leadership II*. These prior and concurrent experiences set the stage and prepared these teachers for the requirements of the Externship.

Externship Components: Needs Assessment

The general goal of the yearlong Externship was to have the teachers conduct needs assessments in the early fall of the mathematics teaching and learning in the teachers' home schools through interviews, discussions, and observations. The needs assessments provided the groundwork and framework for the implementation of a school-based initiative during the late fall and spring which served as the culminating experience for the teachers and their final master's degree projects. By working within the teachers' home schools, the support of the principals and faculty colleagues enabled the teachers to work through this entire process in a safe and supportive environment.

Externship Components: Principal Meetings

The principals of program teachers and the district mathematics supervisors for the three Mathematics Specialists were visited by a Virginia Tech mathematics education faculty member to discuss the design of the Externship requirements and experiences. These meetings occurred before the teachers began the needs assessment process. During the Externship, the teachers interviewed the principals concerning their work and to determine the principals' vision concerning Mathematics Specialists, the mathematical needs within the school building, and concerns or issues that the principals wanted addressed through the Externship process. These meetings not only engaged the principals in the needs assessment process, but also kept the principals connected to the school-based project and helped initiate principal support for the teachers.

Externship Components: Grade-Level Meetings

As a part of the needs assessment, each teacher met with each grade level within their school (and in effect met with all teachers in the building) to discuss hopes and needs for mathematics in the building. These meetings required teachers to take on a leadership role in facilitating these discussions which were focused on specific school-based and grade-level mathematical needs. During these meetings, discussions tended to center on, for example, *SOL* data, classroom data, classroom issues and problems, perceived gaps in students' mathematical understanding, and math content with which students struggled.

Externship Components: Faculty or Department Meeting

As part of the Externship, teachers were required to lead a faculty or department meeting. Through this experience, teachers were able to take on more of a school-based leadership role,

leading a schoolwide discussion of the mathematical needs within the building. These meetings allowed for schoolwide discussions across all grade levels. Therefore, these discussions tended to focus on overall school concerns about mathematics, such as vertical alignment of concepts, schoolwide problem areas as evidenced in overall school *SOL* data, and weaknesses of students from one grade to the next.

Districtwide Initiative

The one shared experience for all teachers in the cohort was designed to provide teachers with the experience of working with a school district initiative. All of the teachers in the cohort were trained on the *Enhanced Scope and Sequence Plus* document [3]. They were then required to plan and provide professional development opportunities for fellow teachers on the appropriate applications and usefulness of this resource. The teachers could work individually or in small groups to design and present this professional development activity to school faculty groups.

Reflections and Class Meetings

Throughout the yearlong Externship experience, eight class meetings were scheduled. During these meetings, activities were designed to support and guide the teachers through this shared experience. Teachers were given very specific writing prompts (see Table 2) to have them reflect on the ongoing needs assessment process and other school-based experiences. These prompts allowed the teachers to reflect on and then share their experiences, issues, and concerns. This mutual sharing provided support for the teachers as they discussed generalities together and brainstormed strategies for dealing with issues or concerns. Even though the nature of each teacher's individual investigation depended on the structure and needs of each individual school, they could share and support each other throughout the process.

Table 2
Writing Prompts

DATE	WRITING PROMPT
SEPT	What are your expectations of the Externship?
OCT	Do you feel prepared for the Mathematics Specialist position? If yes, why? If no, what do you feel you still need to know?
DEC	Reflect on what you have learned about yourself as a Mathematics Specialist through the needs assessment process for your school.
FEB	Have you begun the implementation of your project? If yes, describe one thing that you have done as a part of the implementation and briefly discuss strengths/weaknesses, successes/"wish I had done differently". If no, describe what you plan to do first. As you begin the implementation of your project, discuss your attitudes toward working with students, teachers, or parents, etc. as a "Mathematics Specialist."
MAY	Reflecting on your Externship experience this year, list and discuss two aspects of the Externship that you feel have had the greatest impact on your beliefs about your ability to be a successful Mathematics Specialist. Discuss one thing that you feel would have enhanced your Externship experience and ultimately would have better enhanced your potential to be a successful Mathematics Specialist.

Externship Components: Activity Log/Time Log

Throughout the activities surrounding the needs assessments, teachers kept activity/time logs, documenting all types of activities related to being a Mathematics Specialist in their schools during the year and the amount of time associated with each activity. Once these teachers began the schoolwide needs assessment process, they noticed a shift in their status with other school personnel. Although they were not officially Mathematics Specialists, the role of the teachers informally shifted from classroom teacher to that of a building-level Mathematics Specialist. The principals, fellow faculty, and staff began to treat many of the teachers as Mathematics Specialists; for example, they asked content questions, assessment questions, and sought advice on instructional strategies. Through this documentation process, the teachers could capture these changes in their roles.

Externship Components: Final Project

Based on the needs assessment, each teacher conducted a final school-based project intended to address one of the identified school-based needs in the area of mathematics. This project was to be a systemic program—not a “one-shot” initiative—consisting of a series of meetings, workshops, or instructional sessions that was documented and evaluated over the course of the Externship. Once the specific need was identified, each teacher conducted a literature review to determine appropriate ways to address the determined need. From this research, teachers were able to gain research-based support for the design and structure of their initiative.

Specific projects resulted from the individual needs assessments conducted by the teachers (see Table 3). In three instances, multiple teachers were faculty at the same school so they held the meetings and discussions within the needs assessment process together. In these cases, individual teachers performed individual reviews of the literature and then a joint decision was made as to the most appropriate project for the school. Each individual teacher carried out the implementation of the chosen project. During the implementation of the initiative, teachers were visited and observed by Virginia Tech mathematics education faculty.

Table 3
Title and Description of Teacher Final Projects

<p>Vertical Alignment Through Communication -designed, initiated, and facilitated cross grade-level meetings to discuss <i>SOL</i> requirements, instructional strategies, manipulatives used, and math concepts taught</p>
<p>Using Assessment to Plan Instruction -facilitated collaboration between all grade 2 teachers to implement use of exit slips, anecdotal teacher comments, and other assessment strategies to determine placement of all grade 2 students into groups (remediation, extra practice, and enrichment); used parent volunteers and literacy volunteers to focus students on the created activities designed to meet the needs of each of these groups</p>
<p>Measuring the Alignment of Instructional Delivery in Computation -designed and facilitated a meeting involving all the faculty across two schools (a primary school that feeds into an elementary school) to focus discussions between all grade-level teachers on instructional strategies, vocabulary, algorithms, and manipulatives that currently are being used to teach computation</p>
<p>Parent Nights -designed and implemented parent nights to teach parents the alternative algorithms of partial sums, partial products, and lattice multiplication</p>

<p>Leap into Learning</p> <ul style="list-style-type: none"> -used data to select small groups of students (upper elementary) to attend 18 before school remediation sessions scheduled over 18 weeks -designed curriculum materials and taught the sessions -focused on vocabulary around operations
<p>Building Computational Fluency</p> <ul style="list-style-type: none"> -created and presented a whole-faculty staff development workshop on computational mastery -planned and facilitated Math Mondays, a tutoring session for struggling students using Virginia Tech tutors and math games -focused Learning Lunches on math issues, presented workshop for parent volunteers -coordinated efforts of retired teachers acting as tutors at local trailer parks (created binders of math materials and presented staff development for the tutors)
<p>Manipulative Project</p> <ul style="list-style-type: none"> -surveyed teachers and presented mini-professional development sessions on the topics of fractions, operations, and number sense -designed and presented whole school staff development on manipulative use -compiled a school-wide listing of all manipulatives
<p>Building Computation and Estimation Skills Using Games, Manipulatives, and Songs</p> <ul style="list-style-type: none"> -assisted SOL remediation tutor to identify students using SOL data -designed and led whole-faculty meeting on <i>SOL</i> data results for school to identify areas of concern -planned and facilitated workshop “Fun with Fractions” -facilitated whole school implementation of the use of multiplication fact songs
<p>Building a Mathematical Community One Lesson Study at a Time</p> <ul style="list-style-type: none"> -created, administered, and analyzed faculty survey on areas of concern -used results to choose topic to use for Lesson Study format within two different grade levels (plan lesson, teach, reflect)
<p>From Summative to Formative Data</p> <ul style="list-style-type: none"> -worked with school math teachers to interpret state assessments -facilitated discussions with faculty on the use of formative assessment strategies -assisted teachers in implementing formative assessments (exit slips, observation check lists, reflections, student self-evaluations, student interviews)
<p>The Math Toolbox Program</p> <ul style="list-style-type: none"> -applied and received grant and using these funds purchased manipulatives (cuissenaire rods, rulers, base 10 blocks, and <i>Everyday Math</i> card decks) to create parent kits -designed and presented 5 parent night sessions (games, cuisenaire rods, base-ten blocks, time and money, Achievement Record process) and gave all participating parents a math toolkit
<p>Understanding Math Plus (districtwide software package)</p> <ul style="list-style-type: none"> - designed evaluation process of the software package and use based on funding costs
<p>Teachers + Fractions + Manipulatives = Understanding</p> <ul style="list-style-type: none"> -designed and facilitated 3 sessions of professional development for districtwide faculty -created binder for presentation and to give to participating teachers -observed participating teachers for manipulative use as follow-up to sessions

Externship Components: Final Report

The teachers were required to produce a final report discussing the following topics: 1) the needs assessment; 2) the background literature supporting the final project; 3) the project's implementation plan, anticipated outcomes or research questions, methodologies/procedures, findings/outcomes; 4) overall discussion of the project; and, 5) future plans based on the outcome of the project. The process of writing a final report provided teachers with an authentic experience that would relate to the role of a school-based Mathematics Specialist: i.e., reporting to stakeholders about a school need, the implementation of an initiative to address the need, and the effectiveness of the plan.

Externship Components: Presentation of Final Project

Each teacher was required to deliver a twenty to thirty minute presentation to the cohort and to selected Virginia Tech mathematics and mathematics education faculty. This presentation addressed the background literature, needs assessment, and rationale for the chosen project. The presentation was designed to give each teacher the opportunity to discuss what was done and the results of the project, an evaluation of the project, and the implications for the findings of the project. Similar to the final report, the presentation provided teachers with the experience of communicating the results of a school-based initiative. These presentations also allowed the other teachers in the cohort to experience and follow the process of other needs assessments and initiatives in different school settings with different faculties and different sets of mathematical needs.

Reflections on Externship: Effectiveness

We feel that the Externship's close alignment with the real-world experiences that would be expected in the role of a Mathematics Specialist greatly contributed to the overall effectiveness of the experience. A school-based Mathematics Specialist must collaborate with the principal and teachers to analyze the mathematical needs of the building through interviews and discussions with different groups of teachers, as well as the whole faculty. The required needs assessment provided a structure through which this could happen during the Externship. This structure further ensured that these teachers were engaged in a systematic process of identifying a school need and conducting a review of the literature to determine an appropriate means for addressing the need. Each teacher worked within a unique school setting to determine needs and, therefore, the individual teachers were required to address very real and specific needs as they existed within each specific school.

The continuation of scheduled class meetings throughout the Externship provided support for the teachers as they developed and carried out their individual projects. This support came through opportunities for reflection and discussion with the rest of the teachers in the cohort and the university mentors. This provided a “safety net” for these teachers as they stepped from the classrooms into schoolwide leadership positions.

Reflections on Externship: Strengths

Several aspects of the process emerged as real strengths of the designed Externship experience. One of the major strengths of the experience was that each teacher was immersed in the mathematical needs of one school. They were further required to conduct a review of the research literature to determine an appropriate plan and implementation process to address the identified needs. This individualization and close tailoring of the project to a school’s need brought realism to the entire process—the needs were determined by the stakeholders within each school and the school’s specific need was addressed. Another strength of the Externship was the support it provided for these teachers through continued class meetings. These meetings provided teachers with a safe environment in which to talk through issues and concerns related to their project with cohort members, as well as gain valuable insight from discussions in small and large groups.

During *Mathematics Education Leadership I*, a university faculty member met with each school principal to discuss the coaching process in which the teachers would be involved. This contact was reestablished at the beginning of the Externship with an additional meeting with the principal to discuss the expectations of teachers during the Externship. These contacts made it possible for a strong collaboration between the university and the school, ensuring that the principals felt a strong part of the entire needs assessment process and that the teachers had principal support during these experiences. Principals and other faculty provided additional support for these teachers as a result of the needs assessment and resulting project being so closely aligned with authentic school needs.

Concluding Teacher Comments on Externship Experiences

At the conclusion of the Externship, the teachers were interviewed individually. In response to the final question within the interview protocol, “Is there anything that you gained from the Externship experience that you feel you did not get in any of the other courses in the master’s degree program?” teachers had the following comments:

- “The Externship is a must. I think if we didn’t have that, you almost deprive teachers of the experiences. I think for me the light bulb may not have really come on about what this really is until I was really engaged in it.”
- “You don’t get that experience out working in the school until you are there. You can talk about it in the classroom all you want, but I don’t think you actually understand it until you get out there and work and experience it, like we [did]. We actually had to be a Mathematics Specialist and do those jobs. I think that is when you learn the most.”
- “To integrate it all into realistic classroom settings in the school itself and to work with the teachers and the kids made it more meaningful and made it more realistic and more integrated. It made that wholeness to it and that was what I was hoping for, putting all the pieces together.”
- “It is one thing to talk about and learn about it in your own class, it is another to have to put yourself out to your grade level or to a department or whatever, and another level to do something for a whole school situation. I really think that the Externship has got to be a critical component of the program. There is no way that you can get that kind of experience inside a classroom no matter how good your instructors are. That hands-on experience is so much more valuable than just being told how to go out and do it.”
- “You can read about stuff, but if you don’t do it, you don’t really learn it. I learned that I knew more than I thought I did which has helped, but I also learned that everyone doesn’t know as much as I assumed they do. That was a bigger eyeopener and [something] you cannot learn in a book.”
- “It is the hands-on experience: you can tell me, but until I get out there and do it myself, I am not going to know exactly how it works or exactly what it is supposed to look like. You can’t just sit and listen about it.”

These responses capture the teachers’ feelings about the importance of having time set aside in which to apply the skills and concepts learned. Twelve of the seventeen teachers specifically mentioned the value of being able to apply what they had learned throughout the program during this Externship experience. Based on the comments of these prospective Mathematics

Specialists, it is evident that their experience within the Externship was very beneficial, and provided the opportunity to authentically practice their new roles in a safe and supportive manner.

References

- [1] “Mathematics Specialist Task Force Report,” Virginia Mathematics and Science Coalition Task Force, *The Journal of Mathematics and Science: Collaborative Explorations*, **8** (2005) 5-22.
- [2] L. West and F.C. Staub, *Content-Focused Coaching: Transforming Mathematics Lessons*, Heinemann, Portsmouth, NH, 2003.
- [3] *SOL Enhanced Scope and Sequence Plus*, Virginia Department of Education Training and Technical Assistance Center; Internet: <http://www.ttaonline.org/staff/sol/sol.asp>.

REFLECTIONS ON WHAT YOU HAVE LEARNED: A RAPPORTEUR'S REPORT ON VIRGINIA'S "WHAT WE HAVE LEARNED SYMPOSIUM"

D.B. ERCHICK

*School of Teaching and Learning, The Ohio State University at Newark
Newark, OH 43055*

Abstract

In this Rapporteur's Report on Virginia's "What We Have Learned Symposium" about a statewide Mathematics Specialist program, I discuss emergent topics resulting from presentations and discussion at the Symposium. Topics include defining Mathematics Specialists and coaches, addressing mathematics content in the program, providing ongoing support for Specialists and coaches, and supporting principals. I also provide suggestions, discuss absences in the program, and comment on cautions in revision and needs in dissemination.

Introduction

Serving as the Rapporteur for the "What We Have Learned Symposium" was more than the challenging exercise I expected it to be: it was also an engaging and thoughtful endeavor to learn the details of Virginia's multi-institution, collaborative program for preparing Mathematics Specialists and to do so through the voices of the many and varied participants of the Symposium. The Symposium was an opportunity for participants in Virginia's Mathematics Specialist master's degree programs, offered collaboratively at six Virginia universities, to report on, discuss, and reflect upon their work. My thoughts on the work emerge from my academic background as a mathematics teacher educator with K-12 experience in teaching mathematics in schools, experience working with K-12 teachers, and my own experience as a Mathematics Teacher Leader in an elementary school. That position afforded me the opportunity to do much of the same kind of work the graduates of the Virginia program may do, and also formed the foundation for my current work in Ohio's Elementary Mathematics Specialist Endorsement and Mathematics Coaching Program [1]. As such, my insights and suggestions are grounded both in my experiences and the current directions in mathematics education. I hope they are helpful to the continuance of and ongoing development of the work being done in Virginia and in other states.

Emergent Topics

Over the course of the Symposium and its presentations, I found that five topics emerged and provided room for discussion and growth. First, there was the need for defining the concepts

of Mathematics Specialist and mathematics coach as used in the program and in the research on the program. The second topic involved a discussion of the mathematics content component of the program and how that content is delivered across the institutions in the state that participate in this program. A third topic that emerged from the Symposium was the need for ongoing support for Mathematics Specialists upon graduation from the program and employment in the schools. A fourth emergent topic addressed the importance of the role of building principals in this work of Mathematics Specialists in their buildings. A final topic is addressed as multiple absences that emerged in the discussions. I discuss each of these five topics.

Defining Mathematics Specialists and Mathematics Coaching

In the Virginia program, preparation clearly is intended to support professionals developing an advanced expertise in mathematics education. That expertise includes a deepened knowledge of mathematics content and pedagogy (including learning about equity and diversity issues), as well as experience and support in leadership development. There are no particular roles intended in that training, with the options expected to be broad. Still, much of the language across the participants in the Symposium, and the time-on-task kind of analysis of what the graduates working as Specialists do in their new positions, speaks to the Mathematics Specialists working as mathematics coaches, even though the two labels of specialist and coach are often intertwined and not differentiated from each other in discussion [2]. This condition calls for definitions and answers to related questions. What is a Mathematics Coach? What is a Mathematics Specialist? Do these roles differ? Assuming so, how do the roles differ?

Although the question of defining Mathematics Specialists and coaches emerged in the Virginia work, it is not a question unique to the Virginia program. The question has been raised by others and is a current topic on conference schedules and professional organizations' agendas. Maggie McGatha's inquiry reveals the varied definitions of specialist and coach, and the inconsistency of those definitions as used in the literature [3]. At the time of this writing, nine states offer professional credentials for elementary mathematics specialists, and coaching programs are underway in a number of states. From those initiatives, we are able to begin to define needed features of mathematics coaching programs, but the increased interest, the growing literature, and the lack of consistency in definition move all of us to the same place at which I see Virginia at the time of this Symposium: to define and differentiate specialists and coaches [4].

The Association of Mathematics Teacher Educators (AMTE) has supported a national dialogue on Elementary Mathematics Specialists (EMS). The AMTE Task Force released its

“Standards for Elementary Mathematics Specialists” certification/endorsement at the organization’s 2010 annual conference. In the EMS Standards, the organization describes the foundational knowledge, skills and leadership qualities necessary for the roles and responsibilities an EMS professional may assume [5]. These standards suggest: a prerequisite experience of at least three years of successful mathematics teaching experience; general program guidelines of at least twenty-four semester hours (or equivalent) across mathematical content knowledge, pedagogical knowledge for teaching mathematics, and leadership knowledge and skills; and, a supervised mathematics practicum. The mathematics content knowledge is expected to include both a deep understanding of mathematics in grades K-8, as well as specialized mathematics knowledge for teaching. Pedagogical expectations are grounded in the “NCATE/NCTM Program Standards: Standards for Elementary Mathematics Specialists” and cut across the topics in the categories of learners and learning, teaching, and curriculum and assessment [6]. Leadership preparation should afford specialists the skills and knowledge to work with teachers in a non-evaluative role, but also to know the relevant resources available, work outside the classroom and school to improve mathematics instruction, and use a data-based and informed voice in doing so.

What this means for Virginia’s efforts is that it would serve them well to define their meanings of Mathematics Specialist and mathematics coach. As far as the program is concerned, the definitions would help the state clarify the kinds of work Specialists might do and in turn help constituents see the value of earning the Specialist degree. This would suggest defining the Mathematics Specialist as one having developed particular skills and learned particular knowledge bases, and defining coaching as one of the roles the Specialist might have in a building or school district. However, for the Virginia work, the research may be even more impacted by the definitions. As the Specialists enter into their positions in schools, they define their Specialists’ roles dependent on the context; that is an appropriate approach in many if not all cases. However, when Campbell’s research seeks to define how much time the Specialists spend doing certain tasks and otherwise meeting the needs of the building and its staff, the real difficulties arise. Without fully vetting the tasks that are part of a coach’s role by definition, one cannot accurately account for such time, categorize it, explain, or question it [2].

Mathematics Content in the Project

Clearly the project is grounded in a strong belief in the value of knowing mathematical content, and program structure reflects that foundation. It is just as clear, though, that something very good is happening in the mathematics training. First of all, teachers in the Specialist program—and graduates—continue to want *more* mathematics! I feel confident in saying that for

many and maybe all of us who have worked with elementary teachers, it is truly rare that they ask for *more* mathematics. Undoubtedly, as we do our work, as teachers learn more mathematics and do so in supportive environments, we certainly encounter requests for more mathematics; but, that is typically the result of teachers learning with course instructors under certain conditions. That is what seems to be happening in the Virginia program. Coaches providing feedback to the program, and those reporting and participating in discussion at the Symposium, not only wanted more mathematics, but also appreciated the importance of knowing more mathematics when they teach it. So, not only do they *want* to know more mathematics, but they also *value* the impact of that knowledge for their teaching. This led me to conclude that something very good is happening in the program with respect to mathematics content, and that is a strength of the overall initiative.

Part of the success of the mathematical component of the statewide program is the result of the ways in which the content courses are developed, taught, and coordinated. Mathematics instructors share a common syllabus and, more importantly, share ideas about pedagogy across the state. These instructors remain connected in terms of *how* they teach the content, and that pedagogy discussion allows them to develop a practice that models the pedagogy the Specialists are being asked to learn as well.

Still, the call for more mathematics brings other issues to the discussion, not the least of which is the fact that, for practical reasons, no program can simply add more courses. Doing so might make the program unattainable for some constituents, or doing so might make such a program undeliverable in terms of instructors. In addition, solving the problem of needing “more mathematics” with the simple addition of coursework also removes or at least discourages the need for deep reflection. It is this deep reflection on the content of the program, in their case from a broadly collaborative perspective, which emerged as another asset of the Virginia program. Perhaps it was because the simple solution of adding additional courses was not a practical solution, for the reasons I mention above, that a reflective approach emerged. In any case, that reflective component—one that I might add is an obvious part of the instructors in this project—revealed some interesting insights. One of those was that instructors and program developers recognized that future Specialists’ call for more mathematics needn’t necessarily be about *more*, but, rather, about *different* mathematics. Suggestions from instructors included thinking about applications of the mathematics to provide a context for more depth of understanding of the content. One possibility suggested by the participants is to explore geometry in the context of builders and carpentry [7].

I would add a few ideas to this direction of other ways to address the program participants' needs for more mathematics. One is to think about how one can make better use of the program content in other courses, to save time and build on or form the foundation of teaching about a certain concept. What I mean by this is that instructors, since they are as well versed in the content of courses other than the one they are teaching, can use what I would call a "remember when" strategy in teaching. Although not all of the courses are sequenced, it might be possible to avoid taking time to build on a concept so that one can move to some connected topic that is the content of the course being taught. One approach could be simply to take the students back to another lesson they would have experienced in another course, suggesting they "remember when" they did that work and use that as the start for moving on. This approach would not only help the students see a connectedness across contents, but it would also buy precious time in the current course.

A second suggestion in thinking about addressing participant needs for more content is to think about how it is we determine if the criticism, in this case that the program needs more mathematics, is justified. Certainly, one cannot discount the belief on the part of the participants that they need mathematics. The developing Specialist's voice should be heeded; but, do we hear it literally and add content because they want it? Or do we make the content different so they recognize there is more depth, but not necessarily more content? Or do we think about what else their request might mean? If they have a positive and rich experience with mathematics and want more of it, should we perhaps be teaching that mathematics in even different ways, so that they discover how to learn mathematics on their own—with peers, in private study, and solo exploration—and not just in being taught (and led) in order to become better mathematics teacher educators? I don't have answers to these questions, but I do think the group of instructors and directors in this project is a perfect group, because of the reflective tendencies, to tackle these more-than-academic considerations. It would be important for the program and the field to consider these issues, to better study how the work of Specialists and coaches does indeed need to be content-focused, and how their content knowledge impacts student achievement.

Ongoing Mathematics Specialist Support

In the Virginia program in particular, but more than likely in others as well, ongoing support for the Specialists emerges as a critical issue addressing two particular needs: combating isolation and ensuring continued professional growth. In any context, the Specialist, regardless of his/her role, could be the only person in that role in a particular building, making the job an isolating one. Additionally, the Specialist would not only be alone, but very separate from the

teachers in the building. Even though the Specialist is not an administrator, and is not suggested to be in an evaluative position, s/he is also not considered a classroom teacher. This puts the Specialist in a type of in-between position within the schools' infrastructure. Even if a particular Specialist's role is one that is defined by being a classroom teacher, but teaching only mathematics, s/he is again exists in an in-between space, in part distinguished by and separated from the other teachers by virtue of an advanced mathematics education expertise.

Since the program prepares the participants with competencies for any number of positions, the graduates may be hired in one or more possible roles. To prepare them for that work, with support specific to the job, strands for additional professional development could focus on coaching, curriculum development, professional development, and more. For this and any other context with Specialists in schools, the isolation and limited number of professionals in these roles also suggests that these professionals are not likely to find that support in their schools and perhaps not even in their districts. To counter this situation, I would suggest support of professional gatherings, in the form of local (by region or district) support networks, annual conferences focusing on Mathematics Specialists (and coaching if indeed that is the main role performed by the Specialists), and a statewide professional organization to prevent the isolation understandably related to positions these professionals may hold [2].

The nature of the support, regardless of its purpose being to combat isolation or provide professional development opportunities, could simultaneously address the call by Specialists for more content. Professional development courses where Specialists continue their mathematics learning, perhaps in learning communities as opposed to traditional courses, could meet the need for more (or different) mathematics content while simultaneously helping the Specialists achieve the goal of becoming more independent mathematics learners. Topics that developers and instructors at the Symposium identified as possible additions to the mathematics curriculum, such as discrete mathematics or topics that emerge in their Specialist work, could be worked into the follow-up support. Although content-driven, these courses or other learning experiences can focus on pedagogical moves or particular mathematical processes, all contextualized by mathematics content.

The call for support is, as always in education, limited by funding; however, in this case in particular, technology could help limit costs while connecting the Specialists across the state. Technology-blended courses to reach rural areas of the state, along with a professional "social network," can help program graduates maintain contact and continue professional growth

interactions. This would be more than a website, perhaps password-protected, and serve as a both a storehouse of their own ideas and a source of discussion, support, and ideas for implementing their work. Finally, creating a system—perhaps through the technology, but not limited to that venue—where graduates of the program have the opportunity to mentor new Mathematics Specialists. Again, as with the other suggestions noted herein, this would serve both to prevent isolation and to assure continued professional growth of the Specialists who graduate from the program.

Principal Involvement

From nearly every perspective presented at the “What We have Learned Symposium,” the role of the building principal is recognized as critical. Principal support and understanding of the goals and potential of having a Mathematics Specialist professional in the building impacts the work the teachers do in that building and in many ways determines the success of the initiative. Symposium participants also note how the principal support has already improved over time, by virtue of exposure to the program and having a Specialist in the building. I would suggest taking what is currently more or less happenstance exposure to a level of explicit support by means of any number of opportunities. One is to generate monthly newsletters directed to building principals and distributed to all administrators. These can be distributed electronically to all buildings employing Mathematics Specialists, but also could go to all buildings in the state. If the latter, this would serve to prepare principals and other district administrators *before* they hire a Specialist. Another way to support principals’ understanding of the roles and potential of Mathematics Specialists, I suggest inviting them to classes where their Specialists-in-training and Specialists who have completed the program study, are in attendance. They would be expected to work side by side, and learn in the same manner as program participants. Finally, principals should be offered the following opportunities:

- Specific opportunities for them to learn, ask questions, and participate in their own network;
- A course specifically for their leadership needs in a school with a Mathematics Specialist which could also serve as the start of participation in a principals’ network;
- Continued support within their own professional “social network” which would also serve as a venue for the solicitation of success stories for inclusion in the monthly newsletters; and,
- Sessions specifically for principals in an annual conference for Specialists.

Wondering About Absences

After learning all I did about the Virginia program, I wondered about two topics I did not hear or read about to any great extent. One is the data/assessment connection in the program, and the other is how the program is addressing diversity issues. Although both were mentioned, neither had much of a focus. That absence led me to ask, learn, and develop some suggestions. On the point of data/assessment, a number of questions raised by program and Symposium participants might be addressed. First, the integration of applications into the mathematics content learning could certainly be approached with the use of the program participants' own classroom data, making their experience with data analysis a more authentic one. This approach could also lead to a solution to their request for more mathematics, and the solution of deeper, applied, and different mathematics as opposed to simply including more mathematics. Additional ways to address formative assessment, too, could be embedded in coursework and the program participants' classroom work. Finally, program participants noted a need for revision of the Leadership courses, where Year 1 included too many topics and Year 2 included too few. Both formative and summative assessment topics might easily fill the Year 2 void. I would also suggest that the approach to the data work be developmental, and cut across all of the Leadership courses, perhaps with classroom formative and initial summative assessments in Year 1, deeper and more explicitly focused classroom-based assessments in Year 2, and the beginning of a review of building and district data in Year 3.

Diversity topics are actually less of an absence than less of a visible presence. There is a Diversity course in the program, but my question about addressing diversity issues goes beyond the course. What is the program's stance on diversity—as a program? Its inclusion could be cursory, making the course an obligatory one in order to fill the needs of a funding proposal or underscore mathematics education's continued commitment to serving all students. Or, it could be a foundation where the concepts are explicitly addressed throughout the program. Of course, it could be many things within that range as well. Students, and some instructors, seem to feel that the Diversity course is disconnected and in many ways difficult to teach; they questioned if having the course is appropriate. Certainly, dropping the Diversity course is an option, especially if the program believes the content can be added elsewhere (e.g., across the Leadership courses). I must note here, though, that the Leadership courses do seem to be fairly robust, and if data work is added and other adjustments made, it is not clear where in the Leadership courses the diversity work would go. Still, it is a possibility.

I have fewer suggestions and more cautions about addressing program needs around teaching for and about diversity. One is to consider if the problem is less about the content of the *Diversity* course and more about whether the content of the course or the pedagogies employed in teaching it might be revised. The program seems capable in teaching, for instance, mathematics content in a constructivist manner. So, is the Diversity course designed similarly? That course would have a different type of content than a mathematics course, and perhaps need a revised look at how it might be taught in a more learner-responsive way. That is for the course instructors to consider. Another caution regards the possibility of dropping the course and integrating the content across other courses. Having taught diversity courses myself, I, too, see the value in having an integrated approach. The caution here is not to avoid dropping the course, but instead to be certain that if the program does drop the course, to be vigilant and explicit in making and keeping the content visible through the integration. Make the invisible visible.

Final Remarks

Suggestions and cautions aside, the Virginia program generates good mathematics learning, good teaching (for the Specialists and by the Specialists), thoughtful and compelling results, and is inclusive of multiple voices in the process. In short, a lot of good things are happening here. Dissemination of the work from multiple perspectives would be my last suggestion. As heard throughout the Symposium, coaches, Specialists, supervisors, mathematics content and education faculty, and principals provided perspectives that enriched the discussion. I would encourage all to pursue publication in the many and varied outlets available, both to reach different audiences and provide better understanding and support within those audiences. From the academic perspective, once the program clarifies its definitions of coaching and Specialists, the research results will be more powerful; but, we also need additional perspectives on the data. Within the program rests the potential to generate more case studies, such as the kind of work presented by Whitenack and Ellington, and other qualitative studies to enhance findings from statistical analysis of student achievement and inform the program and the field on teacher development in working with Specialists [8]. This is both a desired and vital part of this kind of work, and the Virginia collaboration is well poised to take on the task.

References

- [1] P. Brosnan and D.B. Erchick, "The Mathematics Coaching Program," The Ohio State University, 2007; Internet: <http://mcp-coaching.osu.edu/Pages/index.aspx>.
- [2] P.F. Campbell and N.N. Malkus, "The Impact of Elementary Mathematics Specialists," *The Journal of Mathematics and Science: Collaborative Explorations*, **12** (2010) [1-29].
- [3] M. McGatha, "What Does the Research Say About Mathematics Specialists & Coaches?" Presentation at the Association of Mathematics Teacher Educators Annual Conference, Orlando, FL, 2009.
- [4] M. McGatha, "The Top Ten List for Mathematics Coaching Programs," Presentation at the National Council of Teachers of Mathematics Annual Conference, Washington, DC, 2009.
- [5] "Standards for Elementary Mathematics Specialists: A Reference for Teacher Credentialing and Degree Programs," Presentation at the Fourteenth Annual Association of Mathematics Teacher Educators Conference, Irvine/San Diego, CA, 2010.
- [6] "NCATE/NCTM Program Standards: Standards for Elementary Mathematics Specialists," National Council of Teachers of Mathematics, 2003; <http://www.nctm.org/standards/content.aspx?id=2978&langtype=1033>.
- [7] R.W. Farley, "Geometry Examples Encountered in Various Everyday Experiences," *The Journal of Mathematics and Science: Collaborative Explorations*, **12** (2010) [85-94].
- [8] J. Whitenack and A. Ellington, "What We Are Learning about the Elementary Mathematics Specialist's Role: Some Reflections about Math Coaching," *The Journal of Mathematics and Science: Collaborative Explorations*, **12** (2010) [31-45].

The Journal of Mathematics and Science: COLLABORATIVE EXPLORATIONS

Volume 12, 2010

PART II: REGULAR JOURNAL FEATURES

Virginia Mathematics and Science Coalition

MATHEMATICS PROFESSIONAL DEVELOPMENT WORKSHOP FOR MIDDLE SCHOOL TEACHERS: CONCEPT VERSUS MEMORIZATION

D. TAYLOR and R.W. FARLEY

*Dept. of Mathematics and Applied Mathematics, Virginia Commonwealth University
Richmond, VA 23284-2014*

Abstract

This article includes professional development topics for middle school mathematics and science teachers from two week-long Urban Teacher Institutes. These Institutes were held at J. Sargeant Reynolds Community College (JSRCC) and its partner institution, Virginia Commonwealth University (VCU), during the summers of 2007 and 2008, and were supported by a grant obtained by Dr. Harriet Morrison (JSRCC). Co-author Dr. Dewey Taylor directed the 2007 workshop, and both authors served as faculty leaders in both workshops. The workshops focused on teaching in an urban environment and “community mapping” (understanding the details of a certain locale to make the teacher more knowledgeable about the environments of both the students and the schools). The community mapping aspect of the workshops was led by Dr. Shirley Key of the University of Memphis. They featured content teaching and applications led by VCU faculty in mathematics, physics, forensics, engineering, mathematics education, and science education. This article focuses on the mathematics professional development strand in the workshop which featured conceptual learning with graphing calculator support as an alternative to the memorization of formulas.

Mathematics Concepts versus Memorization of Formulas

The discovery activities are outlined in the following sections. These activities were investigated either in the teacher workshop sessions or in the plenary presentations of the Urban Teacher Institute.

Find the Formula for the Area of an Ellipse *and Never Forget It!*

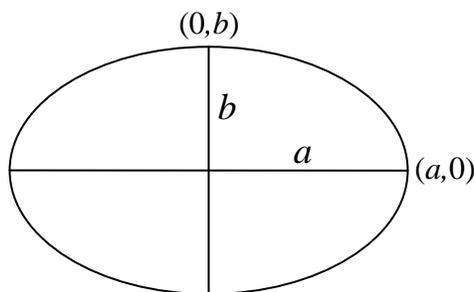
Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with x -intercepts $\pm a$ and y -intercepts $\pm b$

$$a^2 y^2 + b^2 x^2 = a^2 b^2$$

$$a^2 y^2 = a^2 b^2 - b^2 x^2$$

$$y^2 = \frac{b^2}{a^2} (a^2 - x^2)$$

$$y = \pm \frac{b}{a} \sqrt{a^2 - x^2}$$



Using the following integral from calculus, we can compute the area,

$$A = \frac{4b}{a} \int_0^a \sqrt{a^2 - x^2} dx .$$

Let's examine a special case with $a = 3$, $b = 2$

$$A = \frac{8}{3} \int_0^3 \sqrt{9 - x^2} dx$$

Since many teachers will either not have studied calculus or have forgotten it, we use the TI-83 calculator to demonstrate and run several programs which calculate the integral which, when multiplied by 4, gives the area. Setting the window to “zoom decimal,” we first run the program “Riemann” to show the rectangular area which will approximate the actual areas. For this visual, we use $n = 10$ subdivisions to distinctly see the rectangles. Now, we run a program

“Integral” to get a very close area approximation with $n = 500$ divisions. This program will run in slightly more than a minute and will produce the area approximation of 18.84946 when the answer is multiplied by 4. We now ask the teachers to guess whether or not π is a factor of the answer for the area. Most will guess “yes.” So, we now divide the answer by π and obtain 5.9999722 which we interpret approximately as an area of 6. The teachers now guess that 6 is the product of the length of two semi axes, namely $6 = 3 \times 2$. Now, the area in the given case can be guessed to be $A = \pi \times 3 \times 2$ and in the general case to be $A = \pi ab$. We now note that this generalization of the area is πa^2 of the circle which results when $a = b$ is the radius of the circle.

Arithmetic Sum Based on Concept of Average

We motivate this concept by asking: How much money would you make if you **averaged** \$50 per week for four weeks? Most teachers know that the result is $\$50 \times 4$ weeks which equals \$200. We recall that the average per week can represent each actual amount per week which could, for example, have been \$45, \$55, \$51, and \$49, respectively. Next, we pose the following problems and discuss the solutions.

- *Find the average of 2, 4, 9.*
 - The average is $15/3 = 5$ which is the sum of the numbers divided by the number of terms, which defines “average.”

- *Use the average to find the sum of the three numbers.*
 - Since the average 5 can represent each of the 3 terms, the solution is $5 \times 3 = 15$.

- *Find the average of 3, 5, 7, 9, 11, 13.*
 - The average is 8; namely, the sum of the terms which is 48 divided by the number of terms which is 6. We now observe that the sum is “arithmetic” which

means that each successive term is obtained by adding a constant difference (in this case 2) to the preceding term. We note that the average can be found in these arithmetic sums by averaging the first and last term, a fact which will be proved below.

- Use the average to find the sum of the six numbers.
 - The average is $(3+13) \div 2$. Since the number of terms is 6, the sum is $(3+13) \div 2 \times 6 = 48$.
- Prove that the sum S of $1+2+3+\dots+n$ is given by $S = \frac{n(n+1)}{2}$.

PROOF:
$$S = 1 + 2 + 3 + \dots + n$$

$$S = n + (n-1) + (n-2) + \dots + 1$$

By adding the series written forward and backward, we obtain each successive term for $2S$ is $n+1$ so that

$$2S = \underbrace{(n+1) + (n+1) + (n+1) + \dots + (n+1)}_{n \text{ terms}} \quad \text{and} \quad S = \frac{n(n+1)}{2}.$$

General arithmetic series may have the following form of:

$$S = S_1 + (S_1 + k) + (S_1 + 2k) + \dots + S_n$$

$$S = S_n + (S_n - k) + (S_n - 2k) + \dots + S_1.$$

Adding, we obtain:

$$2S = \underbrace{(S_1 + S_n) + (S_1 + S_n) + \dots + (S_1 + S_n)}_{n \text{ terms}} = n(S_1 + S_n).$$

Thus, $S = \frac{n}{2}(S_1 + S_n)$.

Now, $S = \frac{n}{2}(S_1 + S_n) = n \frac{(S_1 + S_n)}{2}$. Many texts use the form $\frac{n}{2}(S_1 + S_n)$ instead of $n \frac{(S_1 + S_n)}{2}$ which tends to force memorization of the formula as opposed to conceptualizing the use of average.

Dividing by n , we obtain $\frac{S}{n} = \frac{S_1 + S_n}{2}$. Since $\frac{S}{n}$ is the definition of average, indeed the average of all the terms is the average of the first and last terms.

So, $S = (\text{number of terms}) \times (\text{avg. term})$ where for arithmetic series:

$$\text{AVG. TERM} = \text{AVG. OF FIRST AND LAST TERMS.}$$

NOTE: The average is also the average of the second and next-to-last term, etc.

$$\text{AVERAGE} = \frac{(S_1 + k) + (S_n - k)}{2} = \frac{S_1 + S_n}{2}$$

However, this rule tends to be less useful since posed questions generally are stated so that the first and last terms are known.

We now pose one final problem: Find the sum of $S = 5 + 10 + 15 + 20 + \dots + 100$. The average term is $105 \div 2$ which implies that $S = (105 \div 2) \times 20 = 1050$.

Concept of the Size of an Acre

Nearly every parcel of land bought or sold in this country is measured in acres. However, very few people, including college graduates, have a concept of the size of an acre. Rather than “looking up” the definition in terms of square feet or square yards, neither of which provides much enlightenment of the size concept, we propose to relate the size to that of a football field. We first have the teachers guess the relative size of an acre by comparison with standard sizes of a tennis court, a basketball court, a soccer field, or a football field. We settle on a comparison with the size of a football field. Nearly everyone will know that a football field is 100 yards long.

Few (a coach or two) will know that the football field is $53\frac{1}{3}$ yards wide. We use the calculator to calculate the football field area to be $A_1 = 5330$ square yards. Using the definition that the area A_2 of an area is 4,840 square yards, we calculate the ratio $\frac{A_2}{A_1} = 1.10$ (approximately).

Thus, a football field is approximately one and one tenth acres so that the size of a football field, not counting the end zones, is a reasonable approximation of an acre.

We now note that farmers approximate an area by “stepping off” seventy yards square. This measure of seventy yards square yields 4,900 square yards to approximate an acre. The ratio of 4,900 square yards to 4,840 square yards yields 1.01 acres, notably accurate to .01. Even if you forget the “farmers’ measure,” the football field measure will provide a good comparison.

Rational Numbers: Converting Repeating Decimals to Fractions

Demonstrating that converting terminating and repeating decimals to their rational number fraction representation will enable middle school mathematics teachers to recognize the equivalence of the definition of rational numbers in either form, namely $\frac{a}{b}$ where a and b are integers and $b \neq 0$, or a terminating or repeating decimal.

Of course, a fraction like $\frac{1}{4}$ can be divided out to yield .25, and conversely, .25 can be written as $\frac{25}{100}$ and reduced to $\frac{1}{4}$. Also, a number like $\frac{2}{7}$ can be divided out to yield $.295714\overline{295714}$ in a repeating decimal form.

However, converting a repeating decimal to a fraction is more difficult. While a repeating decimal like $.35\overline{35}$ can be represented as a geometric series $S = .35 + .0035 + .000035 + \dots$, and summed by the formula

$$S = \frac{35/100}{1 - 1/100} = \frac{35/100}{99/100} = 35/99, \text{ the student must memorize the formula } S = \frac{a}{1-r} \text{ where } a$$

is the first term $\frac{35}{100}$ and r is the common ratio $\frac{1}{100}$. We prefer that the student become familiar with the concept of subtracting the “infinite tails” of decimals as a general way of making these conversions as follows: Let $x = .35\overline{35}$, so that $100x = 35.\overline{35}$. Then, subtracting

$$\begin{array}{r} 100x = 35.\overline{35} \\ - \quad x = 00.\overline{35} \\ \hline 99x = 35 \end{array}$$

and $x = \frac{35}{99}$.

The student will quickly learn to adapt this process of creating and subtracting off “infinite tails” of other such repeating decimals in an example like the following:

$$x = .\overline{123}$$

so that $1000x = 123.\overline{123}$. Then, subtracting:

$$\begin{array}{r}
 1000x = 123.\overline{123} \\
 - \quad x = 000.\overline{123} \\
 \hline
 999x = 123
 \end{array}$$

which implies that $x = \frac{123}{999} = \frac{41}{333}$.

One final example should provide ample concept reinforcement: Let $x = 6.3215\overline{15}$. Then, $100x = 632.15\overline{15}$. Subtracting gives:

$$\begin{array}{r}
 10000x = 63215.\overline{15} \\
 - \quad 100x = 632.15\overline{15} \\
 \hline
 9900x = 62583
 \end{array}$$

Thus, $x = \frac{62583}{9900}$.

Of course, this technique can be used to derive the aforementioned formula for the sum of a geometric series as follows:

$$\begin{array}{r}
 S = a + ar + ar^2 + ar^3 + \dots + ar^n + \dots \\
 rS = \quad ar + ar^2 + ar^3 + \dots + ar^n + \dots
 \end{array}$$

Thus,

$$S - rS = (1 - r)S = a \text{ so that } s = \frac{a}{1 - r}.$$

Once the concept has been mastered, teachers can work in groups to create their own examples. The TI-83 graphing calculator (or other comparable calculator) can be used to check the answers. Setting the “mode key” to the maximum nine decimal places and using the “math key” to convert fractions to decimals and vice versa can provide an ample check.

Solving Systems of Linear Equations

Systems of two linear equations and two unknowns, that have exactly one solution, are taught in eighth grade mathematics and Algebra I. Systems of two linear equations and two unknowns that have either no solution or infinitely many solutions are not discussed. In this workshop, we wanted to explore all the different cardinalities of solution sets to systems of two linear equations and two unknowns, and then generalize these results to larger systems with more than two unknowns. In addition, methods to solve larger systems of linear equations by hand using techniques from linear algebra, as well as how to solve systems on the graphing calculator, were discussed.

To motivate this topic, we started with an activity where we asked the teachers the following questions: Does every system of two linear equations and two unknowns have a solution? Is it possible to find a system of two linear equations and two unknowns that has exactly two solutions? Most teachers guessed the correct answer to the first question, but not the second. The teachers were asked to come up with different graphs to try to illustrate these questions and make conjectures about how many solutions a system of linear equations can have. Even though the teachers were unable to draw two lines that intersected exactly twice, some of them were still hesitant to say that such a system of linear equations does not exist.

We followed up this activity with a worksheet containing three systems of two linear equations with two unknowns, one with exactly one solution, one with no solution, and one with infinitely many solutions. The teachers were asked to solve all three systems using algebra only. Even though the teachers were able to work down to some “end result,” they were not able to interpret their answers correctly. For example, ending up with an equality of $2 = 2$ or $0 = 2$ did not make sense.

In an effort to get the teachers to interpret their results on their own, we asked the teachers to graph each system of linear equations. Suddenly, everyone started to make sense of the answers that they had gotten algebraically. They were able to conjecture that every time a system of linear equations has no solution, one will always end up with an equality at the end that is mathematically absurd; i.e., $0 = 2$. Similarly, the teachers were able to notice that if two equations differed by a nonzero scalar only, then they were in fact the same equation, giving the system infinitely many solutions.

It is easy to remember that every system of linear equations has either exactly one solution, infinitely many solutions, or no solution by simply thinking about a system of two equations and two unknowns. Every pair of lines must either intersect in a point, be scalar multiples of each other (the same line), or be parallel, thus giving exactly one, infinitely many, or no solutions, respectively.

After the teachers were comfortable with this, we moved to systems of three equations and three unknowns. We discussed the different cardinalities of possible solution sets thinking of the three planes modeled by configurations within the classroom. Referencing the planes containing two adjacent walls as plane A and plane B, respectively, and the plane containing the floor as plane C, we could see that planes A and C intersect in the baseline and planes B and C meet in another baseline, while planes A and C meet in the corner line. Now, we can see that these three lines meet in a point where the corner intersects the floor. So, a unique solution is possible. To illustrate the case where three planes intersect in a line, we swing the corner door ajar and let plane D be the plane containing the door. Then planes A, B, and D intersect in the corner line. We can also indicate a no solution possibility which occurs when two planes such as the ones containing the ceiling and the floor are parallel. Of course, all three equations might represent the same plane within which infinite solutions exist. Hence, we need no additional props other than the visualization within the room configuration to model the possible solutions for three linear equations in three unknowns. The teachers were again able to be convinced that every system of three equations and three unknowns was going to have either exactly one solution, infinitely many solutions, or no solution. The teachers were asked to solve the following system:

Starting with $z = 3$ and using back substitution, we get $2y - 21 = -17$ which implies that $y = 2$. Finally, substituting both of these values into the first equation yields $x + 2 + 6 = 9$, hence $x = 1$.

When asked to solve the system

$$\begin{aligned}x + y + 2z &= 9 \\2y + 4y - 3z &= 1 \\3x + 6y - 5z &= 0\end{aligned}$$

by hand, it was clear that some alternative methods would be necessary.

Since we wanted to be able to solve systems that involved any number of equations and unknowns, we introduced the teachers to matrices, Gaussian elimination, and row echelon (and reduced row echelon) form. After learning these ideas, the teachers were led to an understanding of how to use the three elementary row operations:

- 1) Multiply a row through by a nonzero constant;
- 2) Interchange two rows; and,
- 3) Add a multiple of one row to another row.

The teachers were asked to first solve a system of two equations and two unknowns using these three rules, and then solve the above system of three equations and three unknowns. This was enough to notice that the work involved is tedious and that the use of a graphing calculator would be handy.

We used the TI-83 and TI-84 calculators for this lesson, but any comparable calculator could be used. After learning how to input matrices into the calculator, we used the REF and RREF commands under the MATRIX menu to provide the output matrix in row echelon and reduced row echelon forms, respectively. When using RREF, one can read the solution for the system directly from the screen of the calculator for a system that has a unique solution. Similarly, systems that have no solution are easy to recognize as well. We need only look for a row in the output matrix that has all zeros except for the last entry. For example, to solve the system, we calculate

$$\begin{aligned}x + y + 2z &= 9 \\2y + 4y - 3z &= 1 \\3x + 3y + bz &= 0\end{aligned}$$

$$\begin{pmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 3 & 6 & 0 \end{pmatrix} \xrightarrow{\text{rref}} \begin{pmatrix} 1 & 0 & 5.5 & 0 \\ 0 & 1 & -3.5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The last line indicates that $0 = 1$ which shows that this system has no solution, just the same as what happened in the smaller systems that the teachers are familiar with that have two equations and two unknowns. Finally, in the situation where the system has infinitely many solutions, the output matrix will have fewer nonzero rows (meaning nonzero in the reduced row echelon form of the coefficient matrix) than there are variables in the system, as illustrated in the system shown below:

$$\begin{aligned} x + y + 2z &= 9 \\ 2y + 4z &= 1 \end{aligned}$$

$$\begin{pmatrix} 1 & 0 & 5.5 & 17.5 \\ 0 & 1 & -3.5 & -8.5 \end{pmatrix}$$

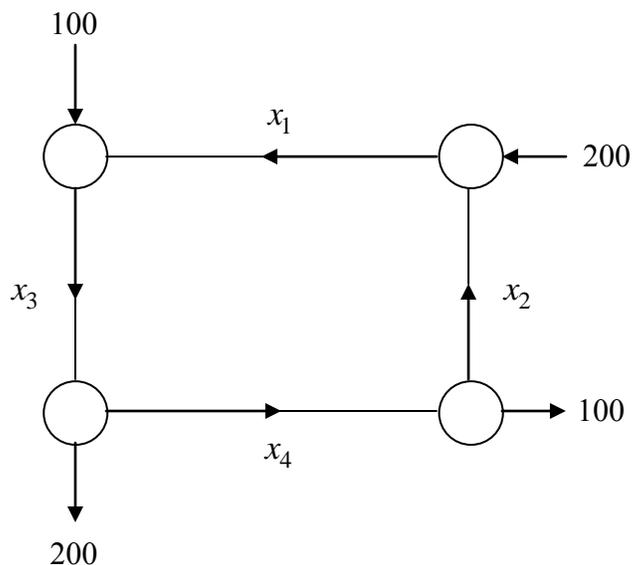
This system can be written now as

$$\begin{aligned} x + 5.5z &= 17.5 \\ y - 3.5z &= -8.5. \end{aligned}$$

Let $z = t$, where t is any real number. Then $y = -8.5 + 3.5t$ and $x = 17.5 - 5.5t$.

The use of the Gaussian elimination has the advantage of allowing one to solve any system of linear equations. Indeed, using the “inverse matrix method” for findings, as is taught for use on the Virginia *SOL*, is limited to systems that have a unique solution.

As an application that could be used for a post-*SOL* activity, we studied traffic flow through an intersection. Given an intersection, we can set up a system of linear equations following the idea that the flow of cars into the intersection has to equal the flow of cars out of the intersection. The teachers can easily set up the system of linear equations, using the calculator to find the reduced row echelon form. Consider the following network of streets.



Starting at the upper left and moving counterclockwise, the system of linear equations that corresponds to this diagram is:

$$\begin{cases} x_1 + 100 = x_3 \\ x_3 = x_4 + 200 \\ x_4 = x_2 + 100 \\ x_2 + 200 = x_1 \end{cases} \Rightarrow \begin{cases} x_1 - x_3 = -100 \\ x_3 - x_4 = 200 \\ -x_2 + x_4 = 100 \\ x_1 - x_2 = 200 \end{cases}$$

The calculator quickly produces the reduced row echelon form of the matrix for this system as:

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & -1 & -100 \\ 0 & 0 & 1 & -1 & 200 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The solution of the system can now be written as

$$x_4 = t, \text{ where } t \text{ is any real number}$$

$$x_3 = 200 + t$$

$$x_2 = -100 + t$$

$$x_1 = 100 + t.$$

From this, we can see that the flow of traffic on the street labeled x_4 controls the flow of traffic in the entire network. For example, if there are 100 vehicles per hour moving along the street labeled x_4 , then the flow of traffic on the remaining streets would be $x_1 = 200$, $x_2 = 0$, and $x_3 = 300$ vehicles per hour. This example also offers the opportunity for the workshop teachers to observe that in this situation, t must be a “whole number” since it represents a number of cars.

Conclusion

Although all of the topics in this paper are not new to teachers, our purpose was to get the teachers to think about math that they may have already known in a slightly different way. Learning how to generalize and think about special cases is an important tool in mathematics and is often difficult for teachers to do.

Acknowledgment

The workshop was supported by the Community Foundation Grant #20070006. Any opinions, findings, and conclusions or recommendations expressed in this material are solely those of the authors and are not necessarily the views held by the funding agency.

UNDERSTANDING THE TRANSITION BETWEEN HIGH SCHOOL AND COLLEGE MATHEMATICS AND SCIENCE

S.A. CULPEPPER

*Dept. of Mathematical and Statistical Sciences, University of Colorado Denver
Denver, CO 80217-3364*

C. BASILE

*School of Education and Human Development, University of Colorado Denver
Denver, CO 80217-3364*

C.A. FERGUSON

*Dept. of Integrative Biology, University of Colorado Denver
Denver, CO 80217-3364*

J.A. LANNING

*Assistant Vice Chancellor for Undergraduate Experiences
Office of Undergraduate Experiences
University of Colorado Denver
Denver, CO 80217-3364*

M.A. PERKINS

*School of Education, Colorado State University
Fort Collins, CO 80526*

Abstract

Mathematics and science education is gaining increasing recognition as key for the well-being of individuals and society. Accordingly, the transition from high school to college is particularly important to ensure that students are prepared for college mathematics and science. The goal of this study was to understand how high school mathematics and science course-taking related to performance in college. Specifically, the study employed a nonparametric regression method to examine the relationship between high school mathematics and science courses, and academic performance in college mathematics and science courses. The results provide some evidence pertaining to the positive benefits from high school course-taking. Namely, students who completed high school trigonometry and lab-based chemistry tended to earn higher grades in college algebra and general chemistry, respectively. However, there was also evidence that high school coursework in biology and physics did not improve course performance in general biology and college physics beyond standardized test scores. Interestingly, students who completed high school calculus earned better grades in general biology. The implications of the findings are discussed for high school curriculum and alignment in standards between high schools and colleges.

Introduction

No matter where in the educational continuum transitions take place (i.e., elementary to middle school or high school to college), systems are needed to ensure that students are prepared and that academic and non-academic factors are considered. In particular, the transition from secondary to post-secondary education is one that is receiving greater attention as data suggests that close to 50% of all college freshman students either fail or are put on academic probation due to poor performance [1]. Further, a large portion of student failure and academic probation is due to failure in mathematics and science courses. Kuh states that “many high school seniors are not prepared academically for college-level work and have not developed the habits of the mind and heart that will stand them in good stead to successfully grapple with more challenging intellectual tasks” [2]. Often, college and university faculty do not consider high school standards to be congruent with college expectations [1, 3]. Hoyt and Sorensen argued that “lax and/or inconsistent standards may create student attitudes, behaviors, and expectations for performance that lead to failure in the college environment” [3].

The literature cites many factors that may affect first-year students’ performance in mathematics and science, including academic preparation, congruence between high schools and institutions of higher education, alignment of secondary education standards and expectations in higher education, and several non-academic factors (i.e., pre-enrollment preparation, social relationships, financial issues, parent background and support) [4]. What is clear from existing data is that the transition between high school and college is not conducive to fostering student success for many graduating seniors. As Conley states, “The two systems—K-12 and post-secondary—evolved in relative isolation. Although each is clearly engaged in education, each has traditionally seen its purposes and goals as distinctly different from the other’s” [5]. Studies show that sufficient academic preparation is essential for success in college. For example, students who complete higher-level mathematics and science courses are more likely to attend college, succeed in college-level mathematics and science courses, and graduate [6-8]. In a review of research in science education, Tai, Sadler, and Loehr specifically point to pedagogical approaches, critical concepts taught, the type of laboratory experience, the degree of lesson structure, instructional technology use, AP science instruction, and mathematical background as factors that predict success in science to varying degrees [9].

Research also suggests a lack of congruence between high schools and post-secondary institutions [1-3, 10]. Brown and Conley found that most state assessments do not align with college and university expectations and the *ACT National Curriculum Survey: 2005-2006* found

that few teachers agreed with college educators on what is important to teach. College professors valued thinking skills over content knowledge while high school teachers valued the exact opposite. The ACT survey suggests that this is due to content knowledge making up most state standards. There is also a body of literature related to mathematical knowledge itself and its impact on science courses. Kuh found that students who do not take upper-level mathematics courses are less likely to complete a baccalaureate degree. Hoyt and Sorensen found that students who receive less than a C- in high school Algebra I, Algebra II, and/or Geometry are more likely to take college remedial math.

Recently, researchers and policymakers are examining the alignment between state assessments and standards, and courses in colleges and universities. In fact, according to the ACT National Curriculum Review, “Inadequate high school coursework may account for at least part of the remediation problem. Too few students may be taking enough high school math (up through Algebra II at a minimum)” [1]. Standards are also problematic as they do not match college and university needs [1]. The *ACT National Curriculum Survey* found the following in its research:

High school teachers are being held accountable to teach students the content and skills listed in state standards. Given those expectations, it is not surprising that our survey found that high school teachers tend to rate more content and skills with higher importance and at greater frequency than do their post-secondary counterparts [1].

Finally, there are also the non-academic factors. Studies have examined behavior and relationship issues. Ferry, Fouad, and Smith found a correlation between family involvement and classes taken in high school [11]. The more parents were involved, the higher level the classes that were taken by their high school-aged children. Nonis and Hudson looked at study habits and found that the amount of time students spend studying is related to the number of mathematics and science classes that students take [12]. However, they found that the strongest predictor for college success is either the ACT (American College Test) or SAT (Scholastic Aptitude Test) score. K. Cockley, et al. compared African-American and Euro-American students and reported differences in self-concept among students which they felt contributed to success in higher education [13].

The aforementioned literature clearly supports the need to better understand the alignment between mathematics and science courses in high school and college. Previous

research in this area has primarily examined the relationship between high school course-taking and post-secondary enrollment and performance on standardized test scores or high school grades on college grades [14-17]. Consequently, additional research is needed to understand the link between high school course-taking and academic performance in college. Accordingly, the goal of this study was to explicitly assess the value of high school course-taking on student performance in freshmen-level mathematics and science courses (i.e., general biology, general chemistry, college algebra, and physics). More specifically, the purpose of this study was to address two central questions. First, how well do standardized test scores predict students' performance in freshmen-level mathematics and science courses? Secondly, what is the contribution of high school course-taking to academic performance in college after controlling for students' standardized test scores? Moreover, a significant relationship between high school and college courses provides evidence for alignment between secondary and post-secondary education.

The following discussion is divided into three sections. The first section discusses the sample, variables, and statistical model used to assess the research questions. The second section presents results from a nonparametric regression and discusses the results in relation to the research questions. The last section discusses the implications of the results and provides concluding remarks.

Methods

Sample—Students for this study attended a public, urban university in the Rocky Mountain region. For purposes of this study, data were collected on students who completed one of four mathematics or science courses, namely general biology, general chemistry, college algebra, and physics, between Fall 2005 and Spring 2008. The study examined data from a total of 2,108 students (i.e., 878 students in general biology, 499 in general chemistry, 482 in college algebra, and 249 in physics).

Variables—Table 1 presents descriptive statistics for the variables of interest. Specifically, the dependent variable, course grade, was a twelve-point scale ranging from zero to eleven to represent letter grades on a +/- scale; e.g., 11 represents an A, 10 is an A-, 9 a B+, 8 a B, etc. Table 1 shows that the average course grades ranged between a C and C+ in Biology (e.g., a mean of 5.6) to a B- in Physics (e.g., a mean of 7.3). We also examined the relationship between students' standardized test scores (as indicated by ACT Mathematics and Science sub-test scores) and credits earned with course performance. In particular, credits earned was an important control variable to account for the fact that students differ in exposure to college classrooms. In

fact, the average student completed biology ($M = 37.1$, $SD = 32.2$), chemistry ($M = 55.0$, $SD = 34.0$), and college algebra ($M = 32.5$, $SD = 29.8$) as sophomores whereas students who completed physics tended to be juniors ($M = 76.7$, $SD = 35.4$).

TABLE 1
Summary of Academic Performance, High School Course Credits, and Demographics by Course

Variable	General Biology ($n = 878$)		General Chemistry ($n = 499$)		College Algebra ($n = 482$)		Physics ($n = 249$)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Course Grade ^a	5.6	4.0	6.3	3.5	6.0	3.6	7.3	3.6
ACT Math	21.4	4.3	22.1	4.1	22.2	3.5	22.6	3.9
ACT Science	21.8	3.7	22.4	3.9	22.5	3.5	22.6	3.8
Credits Earned	37.1	32.2	55.0	34.0	32.5	29.8	76.7	35.4
High School Course Credits ^b								
Algebra	1.68	0.73	1.63	0.73	1.74	0.68	1.54	0.73
Biology	1.52	0.71	1.55	0.77	1.39	0.69	1.49	0.74
Calculus	0.19	0.40	0.23	0.44	0.09	0.28	0.29	0.50
Chemistry	1.05	0.52	1.10	0.53	0.98	0.50	1.05	0.51
Geometry	0.90	0.43	0.88	0.43	0.90	0.40	0.87	0.45
Physics	0.61	0.62	0.68	0.62	0.62	0.61	0.72	0.64
Precalculus	0.31	0.44	0.34	0.44	0.29	0.42	0.37	0.46
Trigonometry	0.26	0.32	0.29	0.33	0.26	0.32	0.30	0.34
Demographics ^c								
Female	0.69	0.46	0.67	0.47	0.62	0.48	0.60	0.49
Asian/Pacific Islander	0.22	0.42	0.24	0.43	0.18	0.38	0.26	0.44
African-American	0.07	0.26	0.05	0.22	0.05	0.22	0.05	0.22
Hispanic	0.16	0.36	0.11	0.31	0.14	0.34	0.09	0.29

Note. *M* = Average, *SD* = standard deviation.

^aCourse grade is a 12-point scale ranging from 0 to 11 where 0 is an F, 1 is a D, 10 is A-, and 11 is an A. ^bHigh school courses credits are taken from students' transcripts and represent the number of years. ^cDemographic variables are coded using zeros and ones, so that statistics represent percentages.

The primary variables of interest were students' completed mathematics and science high school course credits reflecting the goal that was to assess whether standards were congruent with academic expectations within University of Colorado Denver (UCD) classrooms. Table 1 presents the average number of course credits students completed in Algebra, Biology, Calculus, Chemistry, Geometry, Physics, Precalculus, and Trigonometry. For example, students in general biology completed nearly one and a half years of high school biology and students in general chemistry completed approximately one year of chemistry in high school. If students finished a

high school course, it was assumed that they received a D or higher in that course. This study examined courses taken, not performance in such courses, because specific high school course grades were unavailable. Nevertheless, if a student passed a course, according to the state, the student met the standards at a minimum level or higher and we assume the student possessed at least minimum skills.

The bottom portion of Table 1 summarizes the gender and race/ethnicity of the sample by course. Moreover, gender and race were quantified using reference coding. For example, the female variable equaled one for females and zero for males, Asian/Pacific Islander (API) equaled one for Asian/Pacific Islander students and zero otherwise, African-Americans (AFA) equaled one for African-American students and zero otherwise, and Hispanics equaled one for Hispanic students and zero otherwise. Note that White students were the reference group for the race/ethnicity comparisons. Table 1 shows that females consisted of nearly 60% of the students within each class and non-Whites comprised roughly 40% within each class.

Statistical Model—The dependent variable of interest, course grades, was an ordinal measure of course performance that was inherently non-normally distributed. Typically, the use of coarse measures, such as course grade, can result in attenuated, or smaller effects; however, some research suggests that coarse measures are less problematic when the variable has ten or more scale points [18]. Recall that course grades used in this study had twelve scale points.

Given the non-normal nature of course grades, a nonparametric regression technique was employed; specifically, a mixed-rank nonparametric regression model developed by Puri and Sen, for estimating and testing the relationship between each variable and course grades [19]. The Puri and Sen method allows researchers either to rank transform the predictors and dependent variable or simply to use the original variables. In this study, course grades were studied on the metrics defined in the previous section and the predictors were included into the model with their original metrics. Additionally, the relationship between each predictor and the dependent variable can be tested for statistical significance by computing the Puri and Sen test statistic, $PS_p = (n - 1)\Delta R_p^2$, where ΔR_p^2 is the change in R^2 associated with adding variable p to the model. The mixed-rank method considers testing multiple coefficients as multiple tests, so it is appropriate to use a Bonferroni adjustment to control the family-wise type I error rate. In this study, the family-wise type I error was set at a 5% level (i.e., $\alpha = 0.05$), so each PS_p was compared with a test statistic from the χ^2 with one degree of freedom denoted as $\chi_{1-\frac{0.05}{P}, 1}^2$ where P is the number of variables in the model.

Results

Table 2 summarizes the results of the nonparametric regression models for the four classes. The following section discusses statistically significant relationships and presents estimates and statistical significance levels in parentheses. It is important to note that no variables significantly related to performance in physics, so results for physics are not discussed. One explanation for the absence of any significant relationships for physics could be that students who completed physics tended to be upper-level students who had enough distance from high school that pre-collegiate factors did not significantly differentiate performance levels. Consequently, it is possible that the physics results could differ in samples that consist mostly of freshman and sophomore students and future research should explore the extent to which pre-collegiate academic preparation (i.e., test scores and coursework) predicts performance in college physics for freshman and sophomore students. Additionally, there was no evidence that demographic characteristics significantly contributed to predicting course performance beyond academic variables. Consequently, the models discussed in this section did not include gender or race/ethnicity in the models. Overall, the models accounted for 19.2%, 20.7%, and 10.6% of the total student variation in course performance in general biology, general chemistry, and college algebra, respectively.

TABLE 2
Summary of Course Grade Regression Models

	General Biology ($R^2 = 0.192$)			General Chemistry ($R^2 = 0.207$)			College Algebra ($R^2 = 0.106$)			Physics ($R^2 = 0.138$)		
	EST	<i>PS</i>	Sig.	EST	<i>PS</i>	Sig.	EST	<i>PS</i>	Sig.	EST	<i>PS</i>	Sig.
ACT Math	0.19	16.66	**	0.33	29.29	***	0.21	11.39	**	0.12	1.70	
ACT Science	0.08	2.24		-0.01	0.02		-0.07	1.34		-0.02	0.03	
Total Credit												
Hours	0.04	28.59	***	0.02	9.36	*	0.02	5.26		0.02	3.04	
High School												
Course Credits												
Algebra	-0.14	0.44		-0.03	0.01		0.46	2.80		-0.41	1.14	
Biology	0.26	1.81		-0.27	1.63		-0.09	0.15		0.53	2.43	
Calculus	1.03	8.44	*	0.10	0.07		1.46	5.40		0.57	1.07	
Chemistry	0.38	2.04		1.01	10.98	*	0.55	2.72		0.37	0.44	
Geometry	0.13	0.14		0.25	0.37		-0.86	4.07		0.85	1.83	
Physics	-0.13	0.34		-0.22	0.62		0.02	0.01		0.78	3.71	
Precalculus	0.21	0.41		0.96	6.44		0.78	3.66		0.61	1.13	
Trigonometry	0.81	3.59		0.71	1.93		1.51	8.31	*	1.40	3.11	

Note. *PS* denotes the Puri and Sen test statistic where $PS_p = (n - 1)\Delta R_p^2$ for variable p . There was no evidence of course grade differences by demographic characteristics (e.g., gender and race/ethnicity) after controlling for academic variables.

Table 2 shows that several variables significantly related to performance in general biology. Specifically, ACT Math scores ($b = 0.19$; $p < 0.01$), total credits earned ($b = 0.04$; $p < 0.001$), and high school calculus credits completed ($b = 1.03$; $p < 0.05$) positively related to grades in general biology. For instance, after controlling for the other variables in the model, a five-point difference in ACT Math Scores corresponded to nearly a one grade-level difference in biology (e.g., $0.19 \cdot 5 = 0.95$ or about a one-point change in course grade). Additionally, completing thirty semester credit hours (which is equivalent to transitioning from a freshman to a sophomore) prior to taking general biology was associated with approximately a 1.17 point difference in students' course grade level. Stated differently, we would expect a sophomore to earn, on average, one grade level higher than a freshman, a junior to earn, on average, one grade level higher than a sophomore, etc. High school calculus also impacted performance in general biology. In fact, students who completed one year of high school calculus tended to perform one grade level higher than their peers who had no calculus in high school.

Table 2 shows that the findings for general chemistry were similar to those for general biology. Specifically, ACT Math ($b = 0.33$; $p < 0.001$) and total credits earned ($b = 0.02$; $p < 0.05$) related to course grades. Interestingly, ACT Math scores were more related to performance in general chemistry than in general biology and total credits earned were less influential on general chemistry course grades. For example, a five-point difference in ACT Math scores was associated with a 1.65 grade-level difference in chemistry and completing thirty college credit hours was only associated with two-thirds of a difference in chemistry grades. The chemistry findings differed from general biology in another significant way because students who completed chemistry in high school ($b = 1.01$; $p < 0.05$) tended to perform better in general chemistry. That is, students who completed a year of chemistry with a lab in high school tended to earn a grade level higher than students who had no lab-based chemistry in high school.

The third column of Table 2 presents findings for college algebra. In fact, only two variables related to performance in college algebra: ACT Math scores ($b = 0.21$; $p < 0.01$) and high school credits completed in high school trigonometry ($b = 1.51$; $p < 0.001$). Again, a five-point difference in ACT Math scores related to a grade-level difference in course performance similar to general biology. The findings for college algebra also demonstrate the strongest relationship with high school course credits. Specifically, students who completed a year of trigonometry tended to achieve grades 1.5 grade levels higher than their peers with no trigonometry. Recall that Table 1 shows the average grade in college algebra was 6.0, or roughly

a C+. Consequently, one way to think about the impact of completing a year of trigonometry is that students with high school trigonometry tended to earn approximately a B- to a B, on average.

Discussion

High school course selection significantly predicted outcomes in gateway mathematics and math classes, which suggests several considerations for policymakers and education communities. Specifically, this study supports the need to further examine required coursework and would suggest the following recommendations.

First, high schools should better align mathematics and science courses with post-secondary courses to increase the skill level of all students and improve the likelihood of student success in college mathematics and science courses. Second, this research indicates a significant relationship between higher-level mathematics courses and college success. Specifically, high school calculus correlated with higher success in biology, and high school trigonometry correlated with high success in college algebra. These findings not only support the need for sufficient mathematics preparation in high school, but it supports the need to identify college bound students early (in middle school) and to encourage them to take higher-level mathematics courses the last two years of high school.

Finally, success in high school chemistry positively related to success in college chemistry. Many students are not required to take chemistry, only three years of science with a variety of selection around which courses to enroll. Our findings suggest that all high school students who are planning on careers in the sciences, which require success in general college chemistry, should complete a lab-based high school chemistry course. Additionally, the significant relationship between high school chemistry and general chemistry in college suggests that high school and college chemistry courses were more aligned than the other science disciplines studied. That is, students who successfully completed a lab-based course in high school tended to earn better grades in college chemistry. A similar finding was identified for college algebra. Future research should study the alignment between high school and college courses in chemistry and mathematics to learn about best practices to transfer to other disciplines, such as biology and physics.

In conclusion, this study suggests that high school course selection improves college academic success in some disciplines. More research is needed to understand reasons as to why chemistry and college algebra exhibited greater alignment with high school courses.

Additionally, future researchers should examine the alignment between high school and college mathematics and science courses at the state and national levels in an effort to develop effective policies.

References

- [1] *ACT National Curriculum Survey: 2005-2006*, American College Test, Iowa City, IA, 2007.
- [2] G. Kuh, "What Student Engagement Data Tell Us about College Readiness," *Peer Review*, **9** (2007) 4-8.
- [3] J.E. Hoyt and C.T. Sorensen, "High School Preparation, Placement Testing, and College Remediation," *Journal of Developmental Education*, **25** (2001) 26-33.
- [4] W. Gonzalez-Espada and R. Napoleoni-Milan, "The High-School-to-College Transition in Science," in J. Mintzes and W. Leonard (eds.), *Handbook of College Science Teaching*, National Science Teachers Association Press, Arlington, VA, 2006.
- [5] D. Conley, *College Knowledge: What It Really Takes for Students to Succeed and What We Can Do to Get Them Ready*, Jossey-Bass, San Francisco, CA, 2006.
- [6] S.P. Choy, et al., "Transition to College: What Helps At-Risk Students and Students Whose Parents Did Not Attend College," *New Directions for Institutional Research*, **27**(3) (2000) 45-63.
- [7] D.L. Saxon and D.D. Lennex, *College Level Biology Aligned to High School and Middle School Curriculum*, Kentucky State Legislature, Frankfort, KY, 2008.
- [8] National Center for Education Statistics website, Internet: <http://nces.ed.gov/>.
- [9] R.H. Tai, P.M. Sadler, and J.F. Loehr, "Factors Influencing Success in Introductory College Chemistry," *Journal of Research in Science Teaching*, **42** (2005) 987-1012.
- [10] R.S. Brown and D.T. Conley, "Comparing State High School Assessments to Standards for Success in Entry-Level University Courses," *Educational Assessment*, **12** (2007) 137-160.
- [11]. T. Ferry, N.A. Fouad, and P.L. Smith, "The Role of Family Context in a Social Cognitive Model for Career-Related Choice Behavior: A Math and Science Perspective," *Journal of Vocational Behavior*, **57** (2000) 348-364.
- [12] S.A. Nonis and G.I. Hudson, "Academic Performance of College Students: Influence of Time Spent Studying and Working," *Journal of Education for Business*, **81** (2006) 151-159.

- [13] K. Cockley, et al., "Ethnic Differences in the Measurement of Academic Self-Concept in a Sample of African-American and European American College Students," *Educational and Psychological Measurement*, **63** (2003) 707-722.
- [14] B. Bridgeman and C. Lewis, "The Relationship of Essay and Multiple-Choice Scores with Grades in College Courses," *Journal of Educational Measurement*, **31**(1) (1994) 37-50.
- [15] B. Bridgeman and C. Lewis, "Gender Differences in College Mathematics Grades and SAT-M scores: A Reanalysis of Wainer and Steinberg," *Journal of Educational Measurement*, **33**(3) (1996) 257-270.
- [16] B. Bridgeman and C. Wendler, "Gender Differences in Predictors of College Mathematics Performance and in College Mathematics Course Grades," *Journal of Educational Psychology*, **83**(2) (1991) 275-284.
- [17] S.A. Culpepper and E.C. Davenport, Jr., "Assessing Differential Prediction of College Grades by Race/Ethnicity with a Multi-Level Model," *Journal of Educational Measurement*, **46**(2) (2009) 220-242.
- [18] H. Aguinis, C.A. Pierce, and S.A. Culpepper, "Scale Coarseness as a Methodological Artifact: Correcting Correlation Coefficients Attenuated from Using Coarse Scales," *Organizational Research Methods*, **12** (2009) 623-652.
- [19] M.L. Puri and P.K. Sen, *Nonparametric Methods in General Linear Models*, John Wiley & Sons, New York, 1985.

AIMS & SCOPE

Articles are solicited that address aspects of the preparation of prospective teachers of mathematics and science in grades K-12. The Journal is a forum which focuses on the exchange of ideas, primarily among college and university faculty from mathematics, science, and education, while incorporating perspectives of elementary and secondary school teachers. The Journal is anonymously refereed.

The Journal is published by the Virginia Mathematics and Science Coalition.

Articles are solicited in the following areas:

- all aspects of undergraduate material development and approaches that will provide new insights in mathematics and science education
- reports on new curricular development and adaptations of 'best practices' in new situations; of particular interest are those with interdisciplinary approaches
- explorations of innovative and effective student teaching/practicum approaches
- reviews of newly developed curricular material
- research on student learning
- reports on projects that include evaluation
- reports on systemic curricular development activities

The Journal of Mathematics and Science: Collaborative Explorations is published in spring and fall of each year. Annual subscription rates are \$20.00 US per year for US subscribers and \$22.00 US per year for non-US subscribers.

All correspondence, including article submission, should be sent to:

Karen A. Murphy, Editorial Manager

The Journal of Mathematics and Science: Collaborative Explorations

Virginia Mathematics and Science Coalition

Richmond, VA 23284-2014

FAX 804/828-7797

e-mail VMSC@vcu.edu

- For article submission, send one hard copy and one electronic copy of the manuscript.
- The body of the paper should be preceded by an abstract, maximum 200 words.
- References to published literature should be quoted in the text in the following manner: [1], and grouped together at the end of the paper in numerical order.
- Submission of a manuscript implies that the paper has not been published and is not being considered for publication elsewhere.
- Once a paper has been accepted for publication in this journal, the author is assumed to have transferred the copyright to the Virginia Mathematics and Science Coalition.
- There are no page charges for the journal.

Copy editor: E. Faircloth

(Contents continued from back cover)

COACHING: ONE MATHEMATICS SPECIALIST'S STORY C.B. Doyle	111
PROVIDING REAL-WORLD EXPERIENCES: THE VIRGINIA TECH EXTERNSHIP FOR MATHEMATICS SPECIALISTS B. Kreye and J.L.M. Wilkins	119
REFLECTIONS ON WHAT YOU HAVE LEARNED: A RAPPORTEUR'S REPORT ON VIRGINIA'S "WHAT WE HAVE LEARNED SYMPOSIUM" D.B. Erchick	131

PART II: Regular Journal Features

MATHEMATICS PROFESSIONAL DEVELOPMENT WORKSHOP FOR MIDDLE SCHOOL TEACHERS: CONCEPT VERSUS MEMORIZATION D. Taylor and R.W. Farley	143
UNDERSTANDING THE TRANSITION BETWEEN HIGH SCHOOL AND COLLEGE MATHEMATICS AND SCIENCE S.A. Culpepper, C. Basile, C.A. Ferguson, J.A. Lanning, and M.A. Perkins	157

CONTENTS Volume 12

PART I: “What We Have Learned Symposium”

THE IMPACT OF ELEMENTARY MATHEMATICS SPECIALISTS P.F. Campbell and N.N. Malkus	1
WHAT WE ARE LEARNING ABOUT THE ELEMENTARY MATHEMATICS SPECIALIST’S ROLE: SOME REFLECTIONS ABOUT MATH COACHING J. Whitenack and A. Ellington	29
A MATHEMATICIAN’S OVERVIEW OF THE VIRGINIA ELEMENTARY MATHEMATICS SPECIALIST PROGRAM L.D. Pitt	45
HOW TEACHERS LEARN: THE IMPACT OF CONTENT EXPECTATIONS ON LEARNING OUTCOMES J. Reyes	61
EARLY ALGEBRA AND MATHEMATICS SPECIALISTS M.K. Murray	73
GEOMETRY EXAMPLES ENCOUNTERED IN VARIOUS EVERYDAY EXPERIENCES R.W. Farley	83
WHAT COUNTS IN THE PREPARATION PROGRAM OF MATHEMATICS SPECIALISTS AND WHAT LESSONS HAVE WE LEARNED ABOUT WHAT NEEDS TO BE ADDED? S.S. Overcash	93
A WEB OF INFLUENCE: HOW THE MSP PROGRAM HAS SHAPED THE THOUGHTS OF THREE INSTRUCTORS J. Reyes	101

(Contents continued inside)